Efficient Algorithms for Strong Local Consistencies in Constraint Satisfaction Problems

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Abstract

The existing complete methods for solving Constraint Satisfaction Problems (CSPs) are usually based on a combination of exhaustive search and constraint propagation techniques for the reduction of the search space. Such propagation techniques are the local consistency algorithms. Arc Consistency (AC) and Generalized Arc Consistency (GAC) are the most widely studied local consistencies that are predominantly used in constraint solvers. However, many stronger local consistencies than (G)AC have been proposed, even recently, but have been rather overlooked due to their prohibitive cost. This research proposes efficient algorithms for strong consistencies for both binary and non-binary constraints that can be easily adopted by standard CP solvers. Experimental results have so far demonstrated that the proposed algorithms are quite competitive and often more efficient than state-of-the-art methods, being orders of magnitude faster on various problem classes.

Related work

There is a large body of work on strong local consistencies for CSPs both for binary and non-binary constraints. For example, max Restricted Path Consistency (maxRPC) (Debruyne and Bessiere 1997) is a promising local consistency for binary constraints that enforces a higher order of consistency than AC. Despite the strong pruning that can be achieved by strong local consistencies such as maxRPC (Debruyne and Bessiere 1997; Grandoni and Italiano 2003; Vion and Debruyne 2009), such algorithms are rarely used because they are too costly. In the case of maxRPC, algorithms suffer from overheads and redundancies as they can repeatedly perform many constraint checks without triggering any value deletions.

Concerning non-binary constraints local consistencies stronger than GAC, have been proposed in the literature but are not widely used. Typically, such local consistencies take advantage of intersections between constraints to remove inconsistent tuples or to add new constraints. Recently there has been renewed interest for strong consistencies as new ones or refinements of existing ones have been proposed (Lecoutre, Cardon, and Vion 2007; Bessiere, Stergiou, and Walsh 2008; Karakashian et al, 2010). One of the most promising consistencies of this type is max Restricted Pairwise Consistency (maxRPWC) (Bessiere, Stergiou, and Walsh 2008).

Many problems consisting of non-binary constraints are described by table constraints (lists of allowed values). Such constraints are very important in constraint programming as they are present in many real problems from areas such as configuration and databases. As a result, numerous specialized algorithms that achieve GAC on table constraints have been proposed (Lhomme and Regin 2005; Lecoutre and Szymanek 2006, Gent et al. 2007; Lecoutre 2011; Lecoutre, Likitviva-tanavong, and Yap 2012). Since these algorithms achieve GAC, they operate on one constraint at a time. However, these algorithms cannot exploit possible intersections that may exist between different constraints. On the other hand, existing algorithms for maxRPWC and other related strong consistencies are generic and thus very expensive. Hence, any extra pruning that they may achieve by exploiting intersections is outweighed by the cpu time overhead.

Although strong consistencies can be very effective standard methods such as (G)AC are still superior in many problems. Hence it is very interesting to study ways to combine both. One such way is through heuristics that can dynamically select the propagation method to apply during search (Stergiou 2008). In this way adaptive propagation is achieved. In this research we propose and evaluate efficient algorithms for binary and non-binary constraints, focusing on table constraints, that achieve stronger local consistencies than GAC. These algorithms are incorporated in adaptive propagation methods to further improve existing methods for CSPs.

Proposed research and progress

The doctoral thesis is organized in phases, the first six of which have been already fulfilled:

1. Initially, a thorough study of existing literature was required to designate the subsequent research steps. A detailed review of research literature indicated the deficiencies of existing methods and clarified the research direction. The study concerned research work on domain filtering algorithms, strong local consistencies and specialized algorithms. Some conclusions were previously mentioned.
2. The implementation of a basic solver for CSPs, which has been and will be used for the experimental study of the proposed techniques. It is capable of handling both binary and non-binary constraints and implements the basic arc consistency constraint propagation algorithms (AC, GAC), which are the most popular techniques in existing solvers, as well as some alternative techniques that obtain a stronger consistency level, such as max Restricted Path/PairWise Consistency (maxRPC, maxRPWC). As a solver, our system incorporates algorithms for enforcing consistency during search (i.e. MAC) on different branching schemes (2-way and d-way branching), as well as numerous value and variable ordering heuristics.
3. According to the extended theoretical and practical research of existing algorithms conducted in previous phases, we proposed new algorithms for binary and non-binary problems that improve existing ones. This step highlighted the importance of our research since it revealed the relation between the type/predicate of constraints and the level of consistency that was applied. (Balafoutis et al. 2010; Balafoutis et al.2011) propose new algorithms for maxRPC (maxRPC3 and $maxRPC3^{rm}$) that exploit residues of supports and outperform previous ones as well as AC algorithms in some problem classes.
4. A very important class of non-binary constraints is table constraints, thus in (Paparrizou and Stergiou 2012a) we proposed an efficient algorithm for table constraints that is stronger than GAC. The proposed algorithm, called maxRPWC+, is based on the local consistency maxRPWC and allows the efficient handling of intersecting table constraints. Additionally, it can be easily adapted to operate on intensional constraints. Experimental results from benchmark problems demonstrate that maxRPWC+ is clearly more robust than a state-of-the-art GAC algorithm in classes of problems with interleaved table constraints, being orders of magnitude faster in some of these classes.
5. Strong local consistencies play a significant role in adaptive propagation techniques, especially when applied with efficient algorithms such as the ones proposed in this research. Recent work in (Paparrizou and Stergiou 2012b) includes the extension of the techniques proposed in (Stergiou 2008) for non-binary constraints and the solver's ability to choose between more than two constraint propagation techniques (i.e. SAC and maxRPWC), thereby overcoming a major drawback of (Stergiou 2008). As a strong propagator, maxRPWC+ from (Paparrizou and Stergiou 2012a) was used. Evaluation show that the proposed heuristics can outperform a standard approach that applies a preselected propagator, resulting in a robust solver.
6. Our current work is towards improving maxRPWC+, by extending more recent algorithms from the family of STR algorithms (Lecoutre 2011; Lecoutre, Likitvivatanavong, and Yap 2012) to a higher-order local consistency.
7. Soon we will propose strong local consistencies to extend Bounds Consistency (BC) considering the special case of linear constraints following (Zhang and Yap 2000).

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