— Hommage à Guy Perrier —
Sur l'injectivité de l'interface syntaxe sémantique dans les grammaires catégorielles

Ch. Retoré

[joint work with D. Catta R. Moot]

LIRMM, Université de Montpellier & CNRS

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INRIA-Lorraine 1994-1997

- When I met Guy in Nancy, he was preparing this PhD →
- Then studying the properties of the categorial analysis of natural language:
- linear logic, Lambek calculus, Montague semantics
- Denis Bechet, Didier Galmiche, Philippe de Groote, Odile Hermann, Jean-Yves Marion, François Lamarche, Sophie Malecki,...
- This talk: recent work in the style of what we were doing

De la construction de preuves à la programmation parallèle en logique linéaire

THÈSE

présentée et soutenue publiquement le mercredi 25 janvier 1995

pour l'obtention du

Doctorat de l'Université Henri Poincaré – Nancy I

(Spécialité Informatique)

par

Guy Perrier

Composition du jury

Président :	Jean-Pierre FINANCE
Rapporteurs :	Jean GALLIER Claude KIRCHNER Jacqueline VAUZEILLES
Examinateurs :	Jean-Marc ANDREOLI Didier GALMICHE

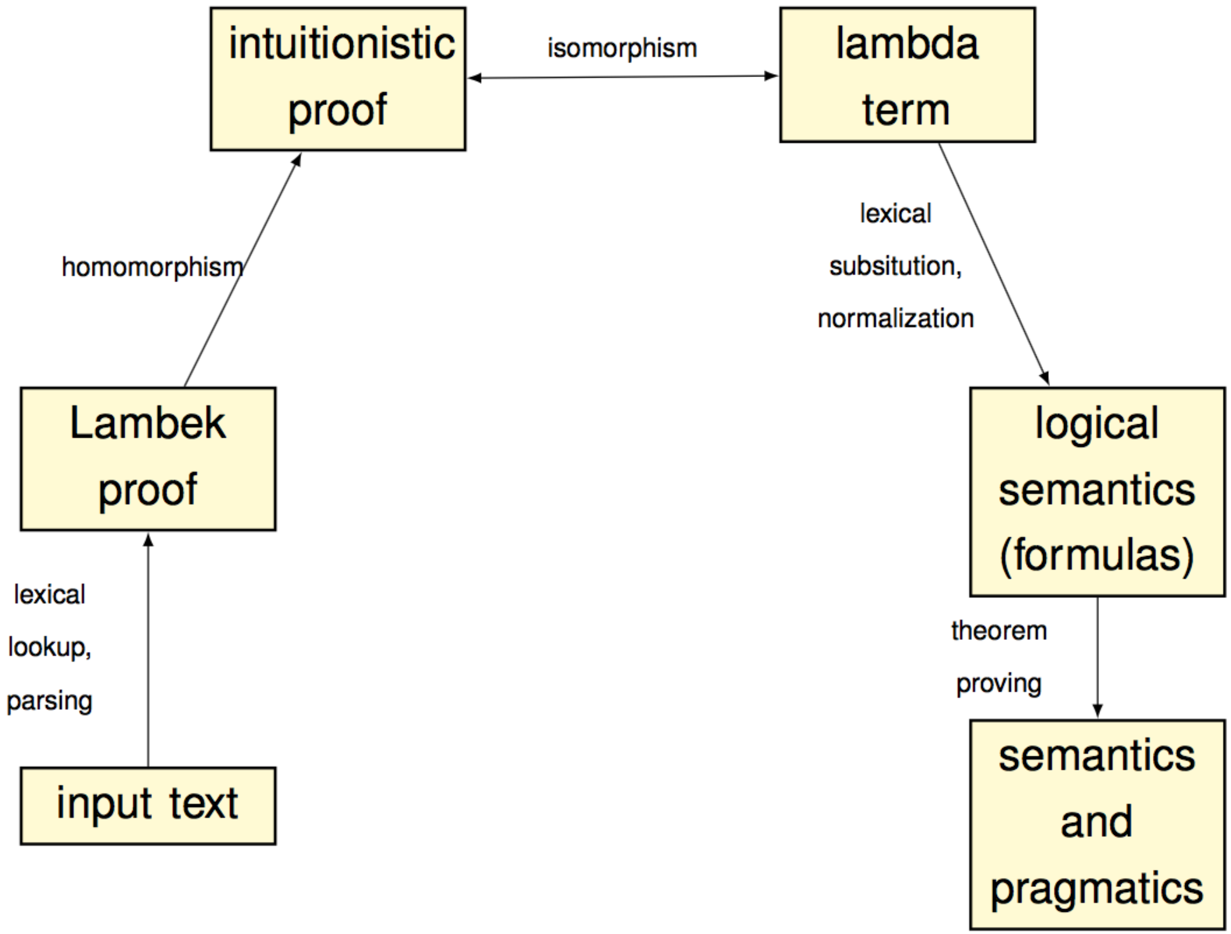
A natural question in categorial grammars

Problem

Imagine that a sentence formed using words $w_1, w_2, \cdots w_n$ has two **different** *syntactic analyses P_1 and P_2. Do those two syntactical analyses yield formally* **different** *semantic representations S_1 and S_2?*

- We will show that this question admits several negative answers if formulated in a naive (but natural) way
- We introduce a relation of dominance between head-symbol (variable or constants) in a λ-term and show that this relation is preserved under β reduction for constant symbols
- We conclude showing that under restricted hypotheses on the semantic lambda terms associated with words the result holds.

Categorial Grammars



The Lambek Calculus L

$$\frac{A/B \quad B}{A}\,[/E]$$

$$\frac{\begin{array}{c}\cdots\cdots[B]^j\\ \vdots\\ A\end{array}}{A/B}\,[/I]_j$$

$$\frac{B \quad B\backslash A}{A}\,[\backslash E]$$

$$\frac{\begin{array}{c}[B]^j\cdots\cdots\\ \vdots\\ A\end{array}}{B\backslash A}\,[\backslash I]_j$$

Two different syntactic analysis

Derivation of ∃∀ reading

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{all}}{(s/(np\backslash s))/n} \quad \dfrac{\text{students}}{n}}{s/(np\backslash s)} \quad \dfrac{\dfrac{[np] \quad \dfrac{\dfrac{\text{wrote}}{(np\backslash s)/np} \quad [np]}{np\backslash s}}{s}}{np\backslash s}}{s}}{s/np} \quad \dfrac{\dfrac{\text{some}}{((s/np)\backslash s)/n} \quad \dfrac{\text{report}}{\text{n}}}{(s/np)\backslash s}}{\text{s}}$$

Derivation of ∀∃ reading

$$\dfrac{\dfrac{\dfrac{\text{all}}{(s/(np\backslash s))/n} \quad \dfrac{\text{students}}{\text{n}}}{s/(np\backslash s)} \quad \dfrac{\dfrac{\dfrac{\dfrac{[np] \quad \dfrac{\dfrac{\text{wrote}}{(np\backslash s)/np} \quad [np]}{np\backslash s}}{s}}{s/np} \quad \dfrac{\dfrac{\text{some}}{((s/np)\backslash s)/n} \quad \dfrac{\text{report}}{n}}{(s/np)\backslash s}}{s}}{np\backslash s}}{s}$$

From L to MLL

$$types ::= \quad e \quad | \quad t \quad | \quad type \multimap type$$

$$s^* = t$$

$$np^* = e$$

$$n^* = e \multimap t$$

$$(A/B)^* = (A\backslash B)^* = B^* \multimap A^*$$

by applying this translation and the Curry-Howard isomorphism we get the linear λ-terms

1. $(w_4 w_5)(\lambda y((w_1 w_2)(\lambda x(w_3 y)x)))$ for the $\exists\forall$ reading
2. $(w_1 w_2)(\lambda x((w_4 w_5)(\lambda y(w_3 y)x)))$ for the $\forall\exists$ reading

From syntax to semantics

- We substitute the lexical meaning for each word. Following Montague, we leave some words analysed, using the constant *students* as the meaning of the word "students", and similarly for "wrote" and "report".
- Using the constants $\forall$ and $\exists$, both of type $(e \to t) \to t$, to represent the universal and the existential quantifier, and the constants $\wedge$, $\vee$ and $\Rightarrow$ of type $t \to (t \to t)$ to represent the binary logical connectives, we can assign the following lambda term to "all" and to "some":

all :
$$\lambda P \lambda Q \forall (\lambda x.(\Rightarrow (P\,x))(Q\,x)) \tag{1}$$

$\lambda P \lambda Q.\ \forall x\ Px \Rightarrow Qx$

some :
$$\lambda P \lambda Q \exists (\lambda x.(\wedge (P\,x))(Q\,x)) \tag{2}$$

$\lambda P \lambda Q\ \exists x P x \wedge Qx$

Semantic lambda terms

$$(\lambda P \lambda Q \exists(\lambda z.(\wedge(Pz))(Qz))report)(\lambda y((\lambda R \lambda S \forall(\lambda v.(\Rightarrow (Rv))(Sv))students)(\lambda x((write\, y)x)))) \tag{3}$$

$$(\lambda R \lambda S \forall(\lambda v.(\Rightarrow (Rv))(Sv))students)(\lambda x((\lambda P \lambda Q \exists(\lambda z.(\wedge(Pz))(Qz))report)(\lambda y((write\, y)x)))) \tag{4}$$

These terms normalize to:

$$\exists(\lambda z.(\wedge(report\, z))(\forall(\lambda v.(\Rightarrow (students\, v)((write\, z)\, v) \tag{5}$$

$$\forall(\lambda v.(\Rightarrow (students\, v))(\exists(\lambda z.(\wedge(report\, z))((write\, z)\, v) \tag{6}$$

A reformulation of the problem

Definition (Syntactic λ-term)

A syntactic λ-term is a β-normal, simply-typed linear λ-term with one occurrence of each free variable in $w_1, \ldots, w_n$ with $n > 0$ — those free variables are the words of some analysed sentence.

Definition (Semantic λ-term)

A semantic λ-term is a β-normal,η-long simply-typed lambda term with constants — it is of type u^* when it represent the meaning of a word of category u.

Assume that the sentence $w_1 \cdots w_n$ has two syntactic analyses P_1 and P_2, when replacing each w_i (a free variable representing m_i in the syntactic analysis that is a linear lambda term) by the associated semantic lambda term t_i in P_1 and in P_2 does beta reduction give different lambda terms , i.e. does one have

$$P_1[w_1 := t_1] \cdots [w_n := t_n] \overset{\beta}{\neq} P_2[w_1 := t_1] \cdots [w_n := t_n] \quad ?$$

A first negative result

Proposition

There exist P_1, P_2 two syntactic λ-terms both of type σ and having the same free variables $w_1, w_2 \dots w_n$, and and there exist $t_1, t_2 \dots, t_n$ n semantic λ-terms such

$$P_1 \overset{\beta}{\neq} P_2 \quad AND \quad P_1[w_1 := t_1] \cdots [w_n := t_n] \overset{\beta}{=} P_1[w_1 := t_1] \cdots [w_n := t_n]$$

Proof.

Take

$$P_1 \equiv w_1((w_2 w_3) w_4) \qquad P_2 \equiv w_1((w_2 w_4) w_3)$$

Moreover take

$$t_1 \equiv \lambda y.k_1 \quad t_2 \equiv \lambda x_1 \lambda x_2((k_2 x_1) x_2)) \quad t_3 \equiv k_3 \quad t_4 \equiv k_4$$

Make the following substitution.

$$P_1[w_1 := t_1][w_2 := t_2][w_3 := t_3][w_4 := t_4] \qquad P_2[w_1 := t_1][w_2 := t_2][w_3 := t_3][w_4 := t_4]$$

Both terms reduces to k_1 □

Refining the analysis

- We have the above negative result because a λ-term may delete something during β-reduction. Hence we restrict the class of semantic λ-terms to *lambda*-I terms only, so β-reduction may not delete anything. This restriction is quite natural when lambda terms that express *word meaning*. Finally, we only consider terms whose head variable is a constant — this technical requirement is admittedly unnatural when dealing with semantics.

Definition (Simple semantic λ-term)

A *simple semantic lambda term* is a β-normal η-long λ_I-term with constants whose head variable is a constant.

Another negative result

Proposition

There exist P_1, P_2 two syntactic λ-terms both of type σ and with the same free variables $w_1, w_2, \ldots w_n$, and and there exist $t_1, t_2 \ldots, t_n$ n simple semantic λ-terms such that

$$P_1 \overset{\beta}{\neq} P_2 \quad AND \quad P_1[w_1 := t_1] \cdots [w_n := t_n] \overset{\beta}{=} P_1[w_1 := t_1] \cdots [w_n := t_n]$$

Proof.

take

$$P_1 \equiv ((w_1 w_2) w_3) \qquad P_2 \equiv ((w_1 w_3) w_2)$$

$$t_1 \equiv \lambda x_1 \lambda x_2 ((k_1 x_1) x_2) \quad t_2 \equiv k_2 \quad t_3 \equiv k_2$$

make the following

$$P_1[w_1 := t_1][w_2 := t_2][w_3 := t_3] \quad P_2[w_1 := t_1][w_2 := t_2][w_3 := t_3]$$

After β-reduction the two terms become β-equal.

□

Well, maybe we should change strategy...

Proposition

There exist P_1, P_2 two syntactic terms, both of type σ, with the same free variables $w_1, \ldots, w_n$ and $t_1, t_2 \ldots, t_n$ n simple semantic lambda terms such that $\forall i \forall j\ 1 \leq i \leq j \leq n$ if $i \neq j$ then the head-constant of t_i is different from the head-constant of t_j.

$$P_1 \overset{\beta}{\neq} P_2 \quad AND \quad P_1[w_1 := t_1] \cdots [w_n := t_n] \overset{\beta}{=} P_1[w_1 := t_1] \cdots [w_n := t_n]$$

Proof.

take

$$P_1 \equiv w_1(\lambda x \lambda y((w_2 x) y)) \qquad P_2 \equiv w_1(\lambda y \lambda x((w_2 x) y))$$

where $x : e, y : e, w_2 : e \rightarrow (e \rightarrow t), w_1 : (e \rightarrow (e \rightarrow t)) \rightarrow t$. Take

$$t_1 \equiv \lambda P(k_1((Px)x)) \quad t_2 \equiv (\lambda z \lambda y((k_2 z) y))$$

where $P : (e \rightarrow (e \rightarrow t)) \rightarrow t$, $k_1 : t \rightarrow t$, $k_2 : e \rightarrow (e \rightarrow t)$ and x, z, y are of type e. And make the following substitution

$$P_1[w_1 := t_1][w_2 := t_2] \quad P_2[w_1 := t_1][w_2 := t_2]$$

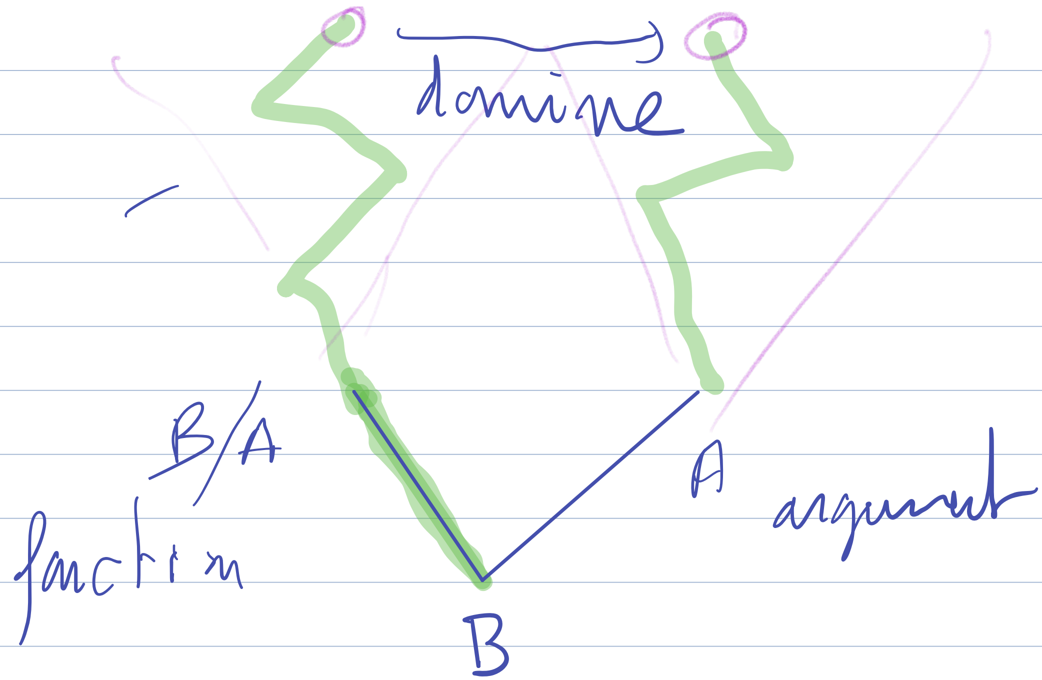
Dominance

Definition

In a term M, occurrences of constants and variables are endowed with a dominance relation as follows.

- If the term is a constant or a variable there is no elementary dominance relation.
- If the term M is a sequence of applications $T_0 T_1 \cdots T_n$ the elementary dominance relations are the union of the ones in each of the T_i, as well as the following additional relations: the leftmost innermost normal sub-term's R head-variable (or constant) h of the term T_0 dominates all head variables (that possibly are constants) of all the leftmost innermost normal sub-terms of the T_i's.
- If the term M is a sequence of abstractions $\lambda \vec{x}.\ t$ (t is not itself an abstraction) then the dominance relations are the ones in t.

The occurrence of a variable or a constant x elementary dominates the occurrence of variable or constant y is written $x \lhd_1 y$ and $\lhd$ stands for the transitive closure of $\lhd_1$.



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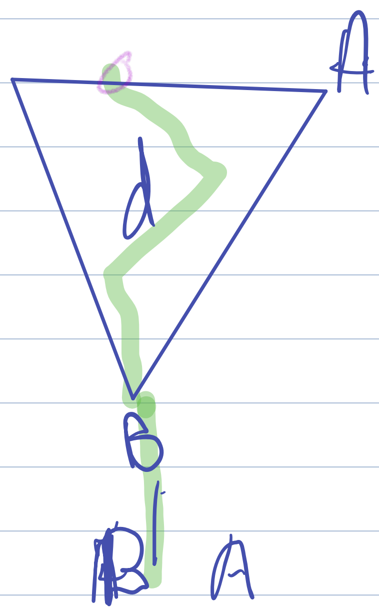

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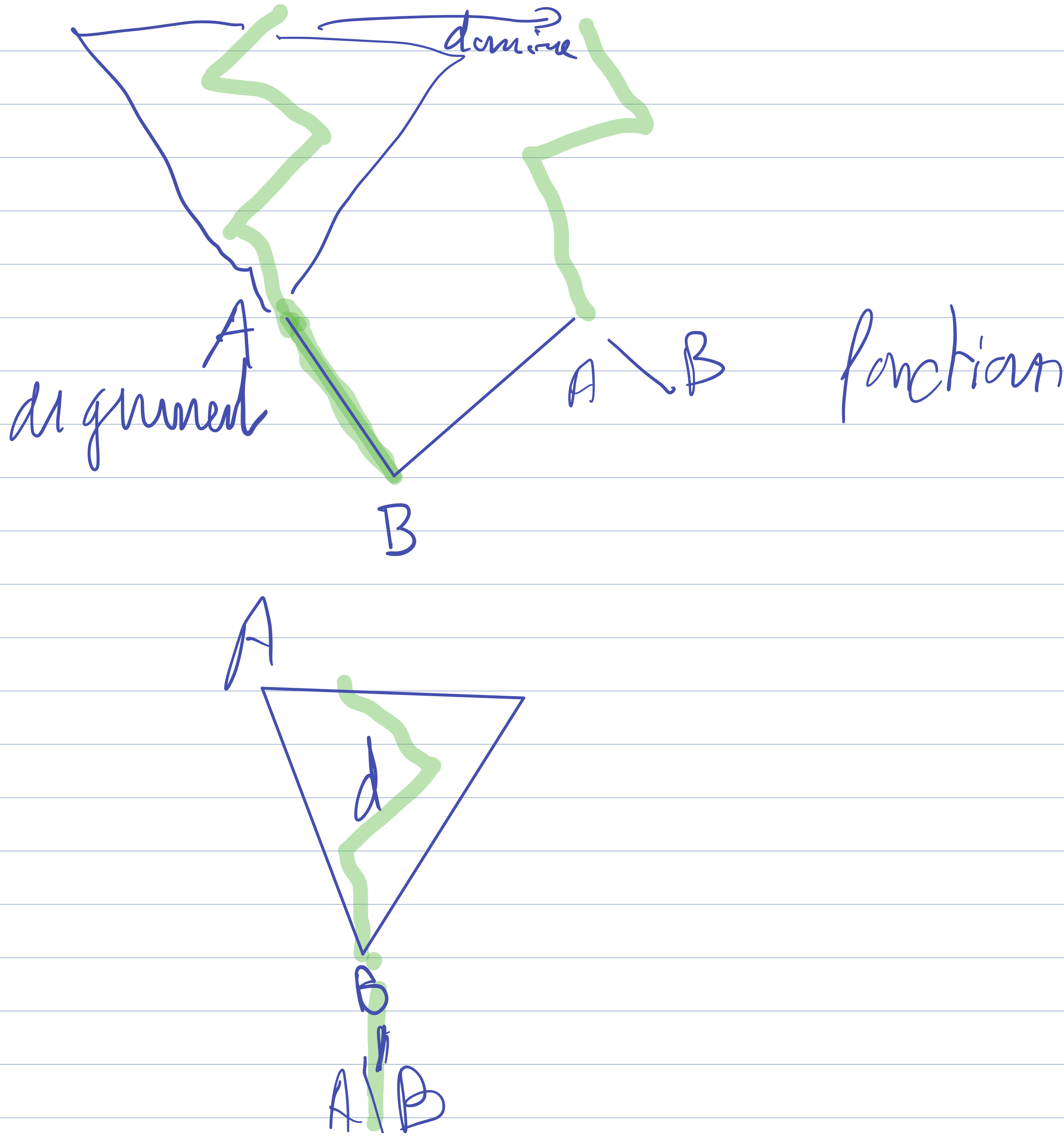
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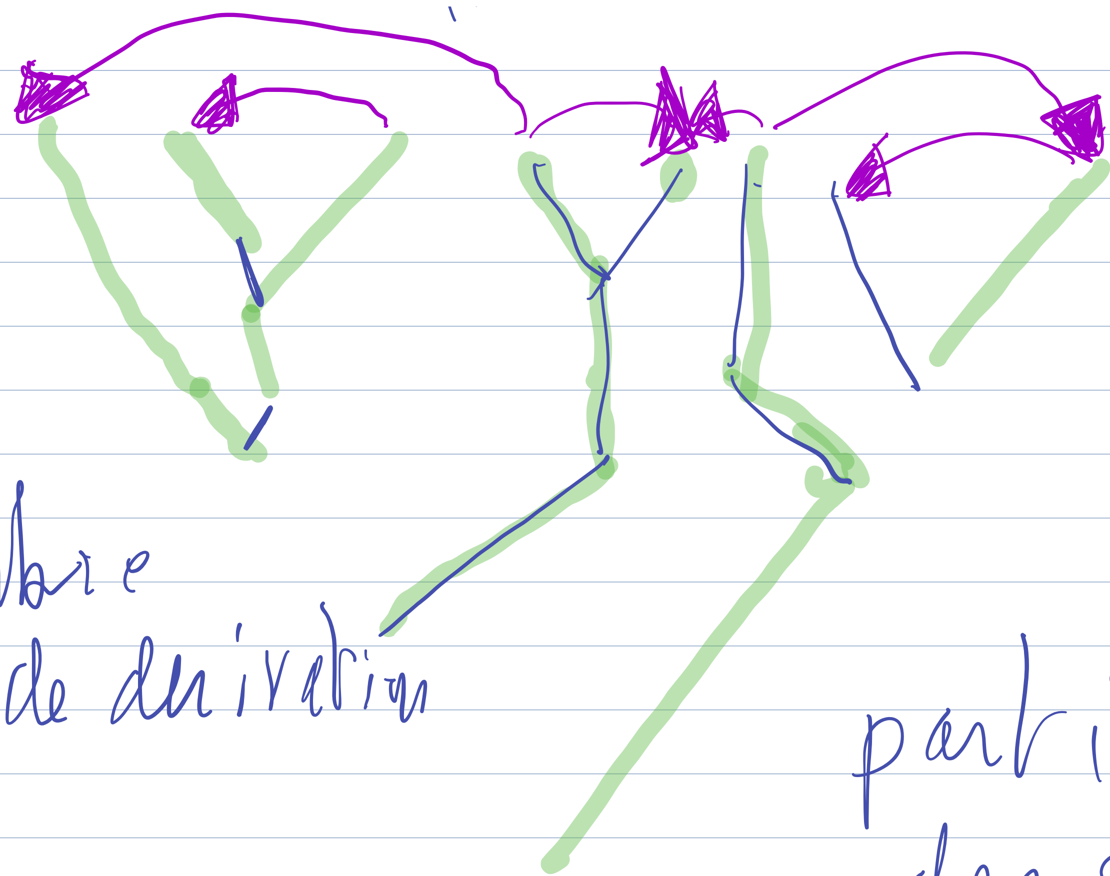


arbre de dérivation

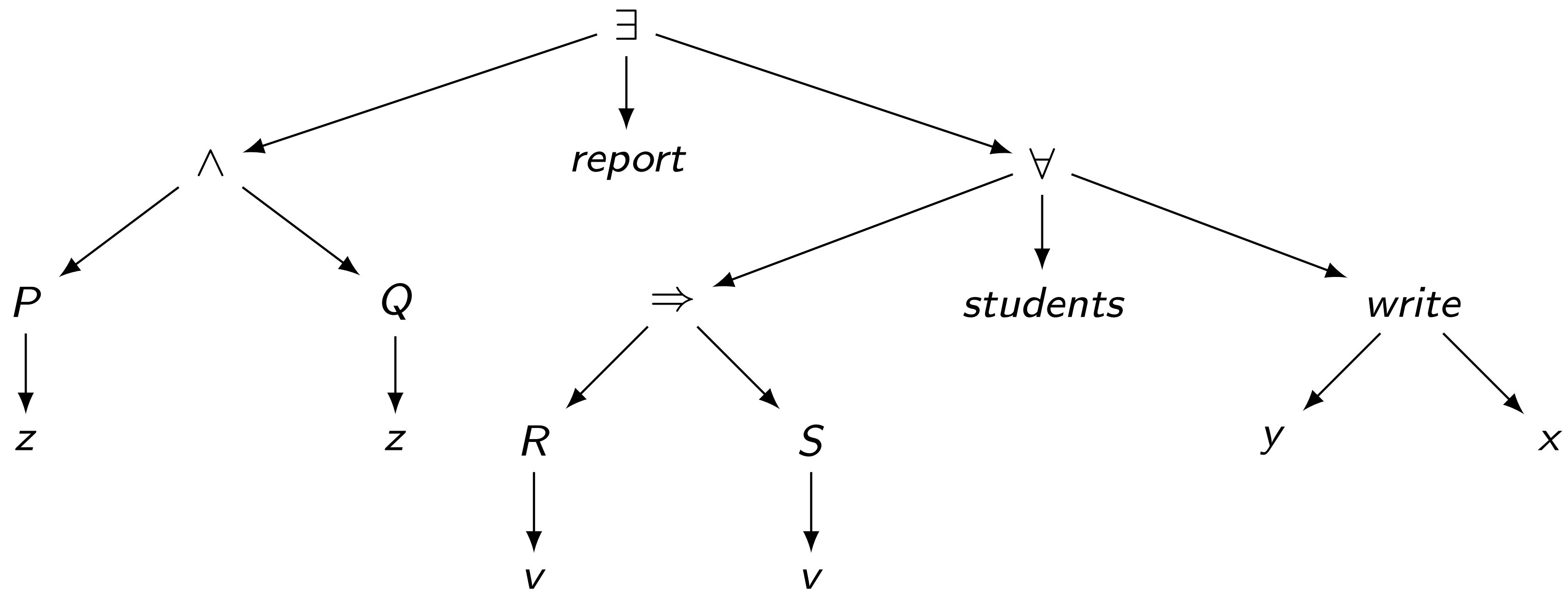
partition des sommets de l'arbre

An example

The λ-term

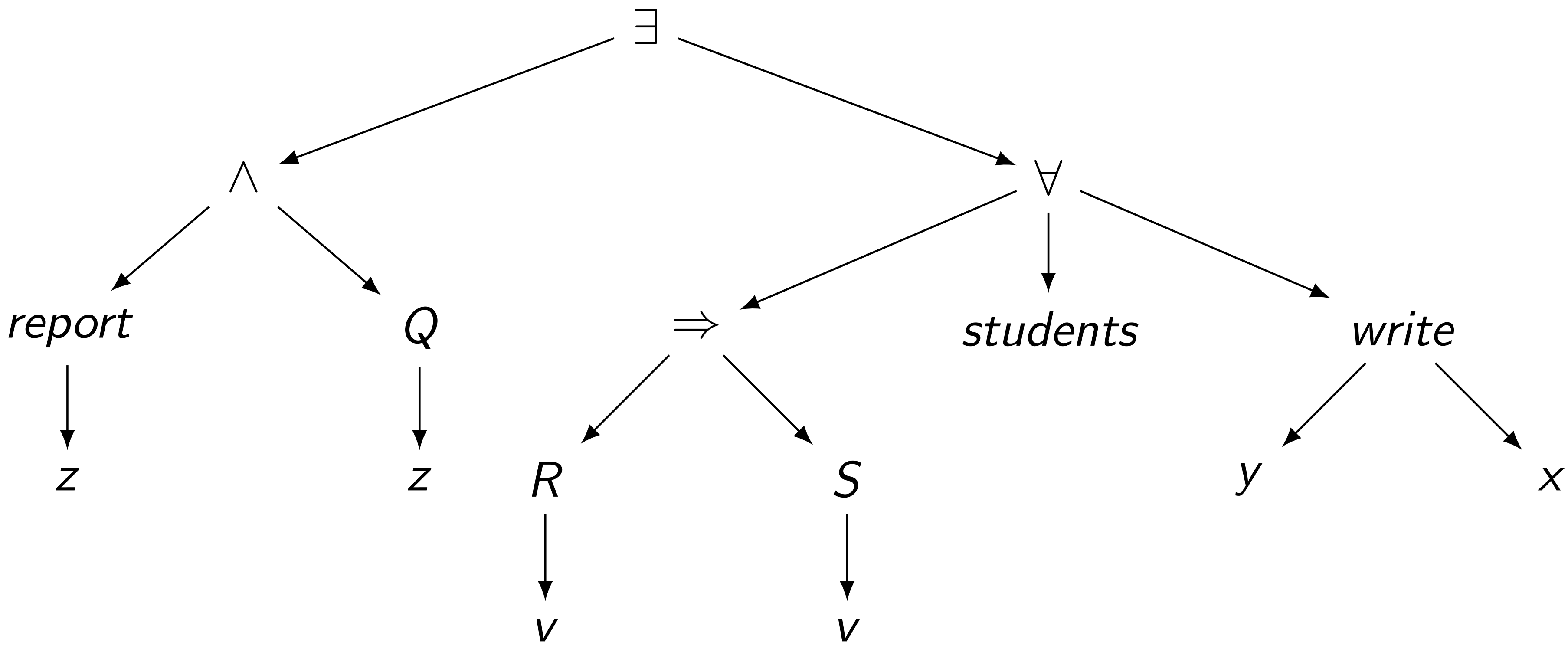
$$(\lambda P \lambda Q \exists (\lambda z.(\wedge (P z))(Q z)) \mathit{report})(\lambda y((\lambda R \lambda S \forall (\lambda v.(\Rightarrow (R v))(S v)) \mathit{students})(\lambda x((\mathit{write}\ y) x))))$$

defines the following dominance relation



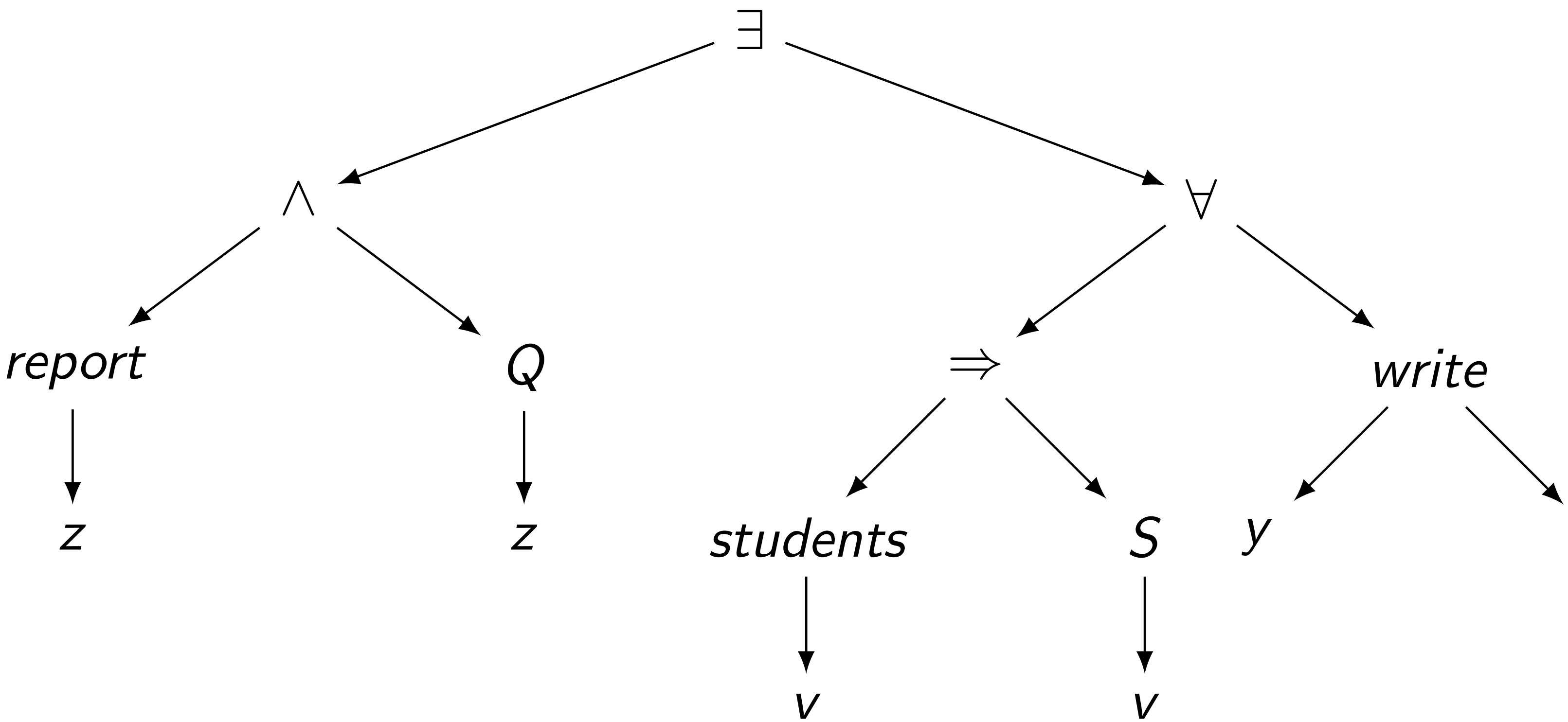
Dominance through β-reduction

$$(\lambda Q \exists (\lambda z.(\wedge (\mathit{report}\ z))(Q z)))(\lambda y((\lambda R \lambda S \forall (\lambda v.(\Rightarrow (R v))(S v))\mathit{students})(\lambda x((\mathit{write}\ y)x))))$$



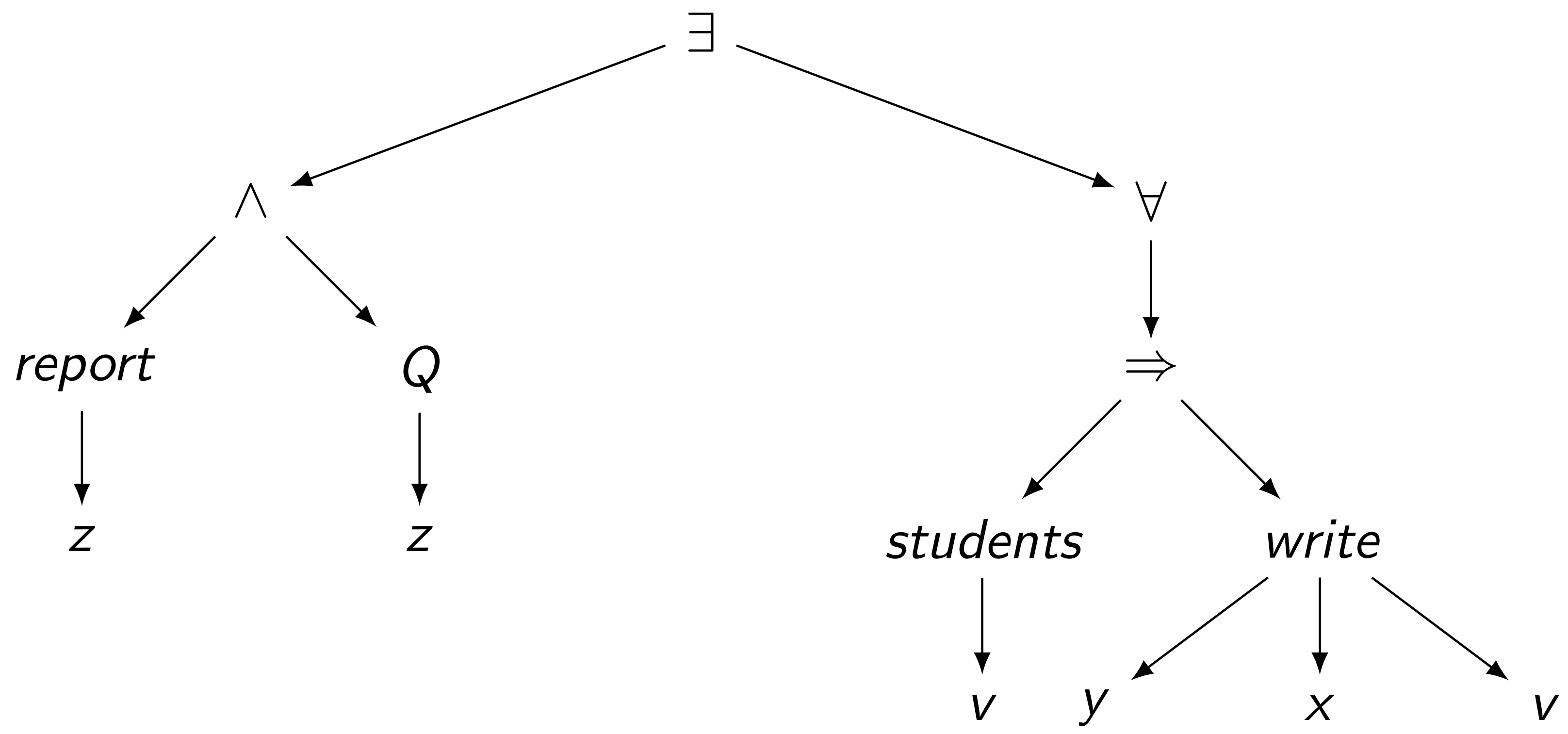
Dominance through β-reduction

$$(\lambda Q \exists(\lambda z.(\wedge(report\ z))(Qz)))(\lambda y((\lambda S \forall(\lambda v.(\Rightarrow\ (students\ v))(Sv)))(\lambda x((writey)x))))$$



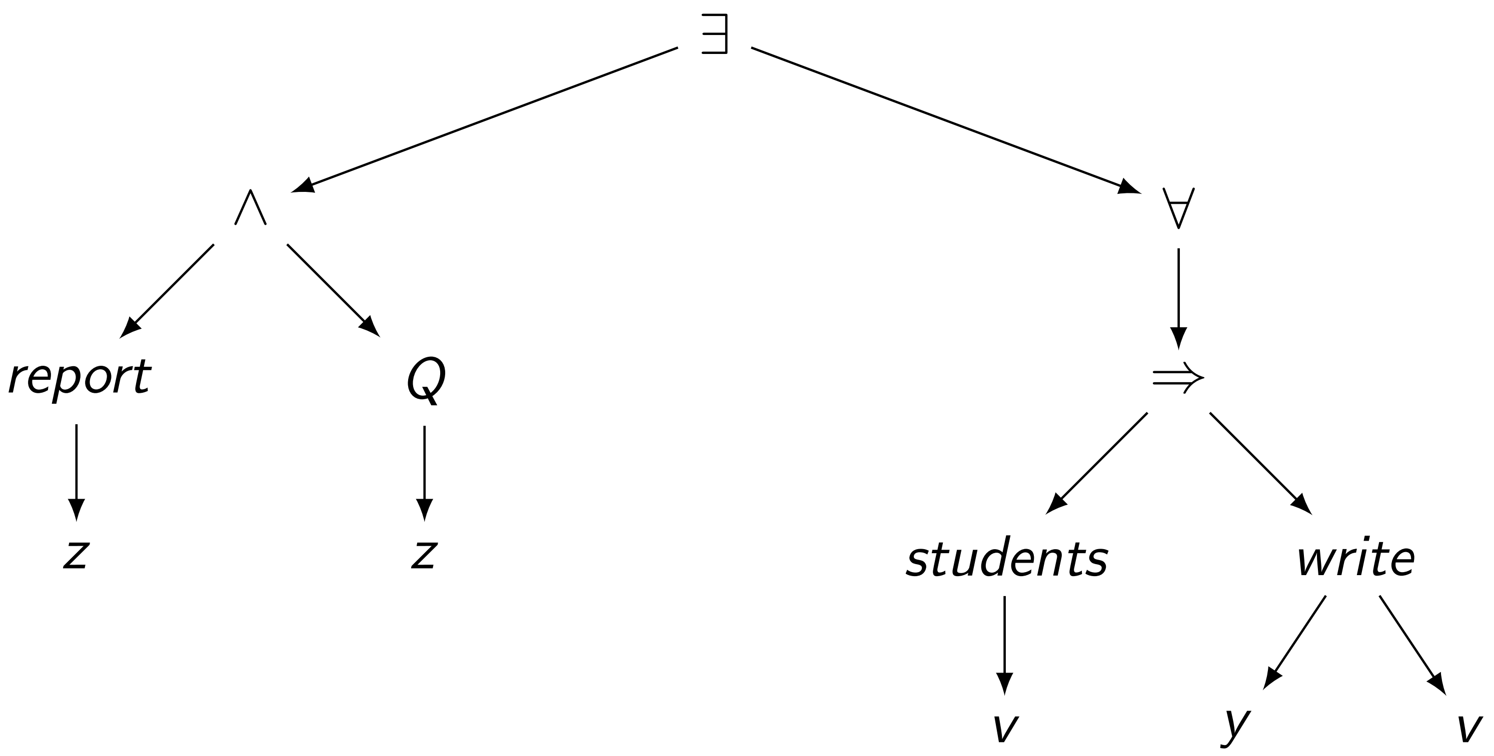
Dominance through β-reduction

$$(\lambda Q \exists (\lambda z.(\wedge (report\ z))(Qz)))(\lambda y(\forall(\lambda v.(\Rightarrow (students\ v))((\lambda x((write\ y)x)))v)))$$



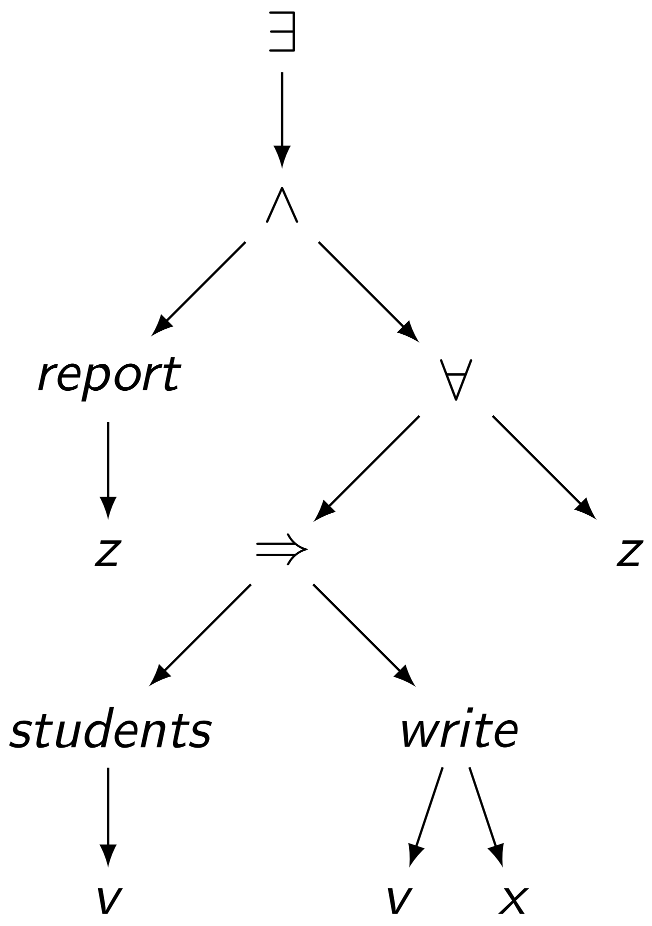
Dominance through β-reduction

$$(\lambda Q(\exists(\lambda z.(\wedge(\mathit{report}\ z))(Qz)))(\lambda y \forall(\lambda v(\Rightarrow (\mathit{students}\ v))((\mathit{write}\ y)v)))$$



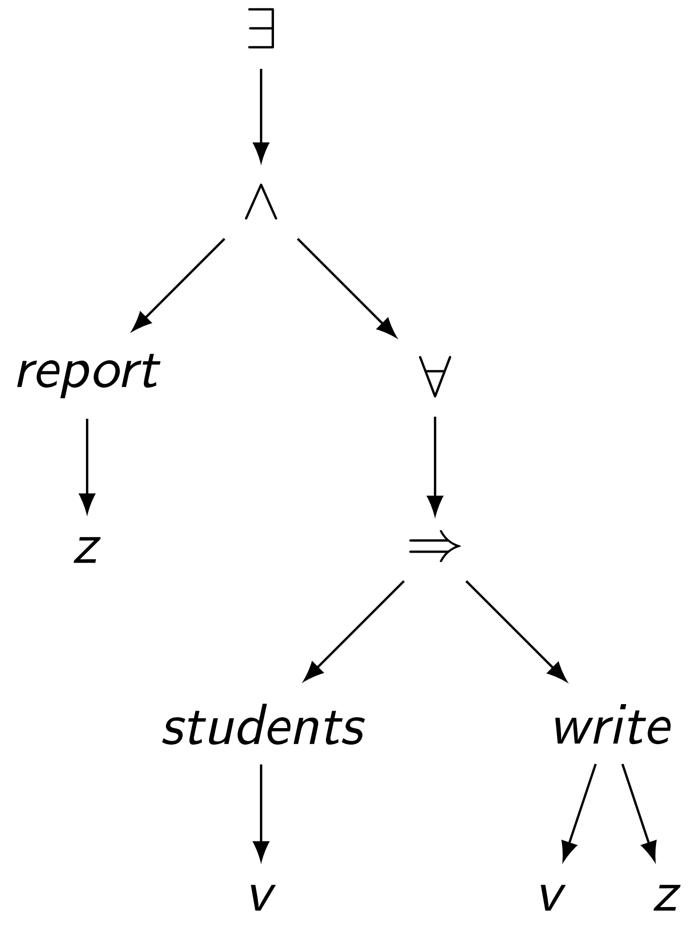
Dominance through β-reduction

$$(\exists(\lambda z.(\wedge(report\ z))(\lambda y(\forall(\lambda v((\Rightarrow students\ v)((write\ y)v))))z)))$$



Dominance through β-reduction

$$(\exists(\lambda z.(\wedge(report\ z))((\forall(\lambda v(\Rightarrow((students\ v))((write\ z)v))))$$



- Remark that on the above term $\exists \lhd \forall$ after each step of β-reduction. This is indeed a general property. We first state two easy proposition

Proposition (1)

Let $(\lambda x A)B$ be a redex where $\lambda x A$ is in normal form. Suppose that K is in $\lambda x A$ and k' is in B. $k \lhd k'$ iff k is the head constant of $\lambda x A$

Proposition (2)

Let P be a syntactic lambda term with words $w_1, \ldots, w_n$. Let t_i be the corresponding simple semantic lambda terms with head constant k_i. If $w_{i_0} \lhd w_{i_1}$ in P then $k_{i_0} \lhd k_{i_1}$ in $P[\vec{w} := \vec{t}]$.

Dominance preservation

se : $\lambda P(\_)\ \lambda x\ P(x,x)$

Proposition (Dominance preservation)

Let U be a typed lambda I term including two occurrences of constants k and k' such that $k \lhd k'$ in U. Assume $U \xrightarrow{\beta} U'$. Then each trace k_i of k is associated with a set of occurrences $k_i'^j$ of k' in U' with $k_i \lhd k_i'^j$ in U' — the sets $K_i' = \{k_i'^j\}$ define a partition of the traces of k'. In particular there never is a relation the other way round after reduction: $k_i' \not\lhd k_i$ in U' for all i.

Proof.

Wlog we show that dominance is preserved for one step of innermost β. Consider the redex $(\lambda x.A)B$ in U and suppose that k and k' are somewhere in the redex (otherwise the result is trivial). We consider two cases

1. k is in $\lambda x A$ and k' is in B. We know that $k \lhd k'$ imply that k is the head-constant of the leftmost innermost normal subterm of A. This imply that $A[x := B]$ has k still . Consequently the (possibly many) instances of k' in $A[x := B]$ are dominated by k
2. k, k' are both in $\lambda x.A$ and we have that $k \lhd_1 x \lhd_1 k'$. Since we are considering innermost reduction $\lambda x A$ and B are normal terms. This imply that B has a head variable or constant h in $A[x := B]$ and for the definition of the dominance relation $k \lhd_1 h$ moreover $h \lhd_1 k'$

□

Corollary

Assume two syntactic terms P_1 *and* P_2 *give opposite dominance relation between free variables,* $u \triangleleft u'$ *in* P_1 *and* $u' \triangleleft u$ *in* P_2*. Whatever the semantic lambda terms substituted for* u *and* u' *with different head constant* k *and* k' *are, the associated logical forms will be different.*

Conclusion

- We have shown that in order to prove our result for linear lambda terms we should take some very strong hypothesis. We however believe that given two different D_1, D_2 normal proof in *Lambek* containing the same undischarged hypothesis $w_1 \cdots w_n$ they will give us two linear lambda D_1*, D_2* terms in which $w_i \triangleleft w_j$ in D_1* and $w_j \triangleleft w_i$ in D_2*. This is work in progress!