Formal proofs and natural language

Christian Retoré – LIRMM (Université de Montpellier, CNRS)

joint work with:

David Catta – LIRMM (Université de Montpellier, CNRS)

Alda Mari – IJN (CNRS ENS EHESS, Paris)

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A Foreword: semantics, argumentation, coherence

A.1. Little / A little

Consider the following dialogue:

(1) Alda — Could you lend me some money?

(2) Bob — Sorry, I can't.

(3) Alda — Why?

(4) a. Bob — * I have a little money.

b. Bob — I have little money.

Although both *little* and *a little* both mean *not much*.

A.2. Tout / chaque

(5) Alda — Tout chien a 4 pattes.

(6) Bob — Pas Rex.

(7) Alda — Il a eu un accident.

The exception does not refute the ”tout” sentence.

(8) Alda — Chaque chien de l’élevage d’à côté aboie jour et nuit.

(9) Bob — Pas Rex.

(10) a. Alda — Ah oui j’oubliais Rex, tu as raison.

b. Alda — Mais non, Rex est mon chien.

The ”chaque” sentence is refuted — or ... two domains.

A.3. Moral

Examples above show at least two things:

- the argumentative aspect of a sentence participates in the coherence of a discourse or dialogue,
- two expressions may have similar denotations but different argumentative uses

A.4. Standard semantic view

The meaning of a statement S is
the collection of the models in which S is true.

models: possible worlds in a Kripke structure

Cognitive / computational problems:

- Infinite non enumerable set of possible worlds.
- Every model is itself infinite non enumerable.
- Leaves out argumentative aspect of meaning.

B Proof theoretical semantics

Natural language sentences $\sim$ logical formulas
(this sometimes makes sense, e.g. in maths)

So: what exists in (mathematical) logic?

B.1. Intuitionism: $[[A]] = \{d | d \vdash A\}$

BHK Curry-Howard categorical interpretations
Martin-Löf Type Theory $\rightarrow$ Homotopic Models

Meaning of a formula : its (formal) proofs.

Proofs have a computational content.

Proof reduction or cut-elimination:
$\rightarrow$ only normal proofs proofs in $[[A]]$?

Impossible for classical logic.
All proofs of a formula reduce one to another.

B.2. Limitations

1) First order logic and
2) extra logical axioms
easily express theories (beyond arithmetic)

but some formal properties of proofs are lost:

1. First order: subformula property is weaker.
2. Axioms: normalisation is weaker.

C Formal and informal justifications for natural language sentences

C.1. Meaning as justifications

Transposition of proof theoretical semantics.

Formula	F	$\rightarrow$	formal proofs of	F
Sentence	S	$\rightarrow$	justifications of	S

Related to text entailment: is a sentence or paragraph consequence of another?

Mathematical practice use natural language.

Observe that when learning to read children are asked text entailment tasks.

C.2. Justification as formal proofs

(if) a sentence corresponds to logical formulas, then justifications = proofs of those formulas.

Which logic?

- Intuitionistic logic (for having several non equivalent justification) or modal logic ?
- First order / many sorted / Type Theory
- Axioms
  - for word meaning (dictionary definition)
  - for observations
  - for opinions

Subjective axioms and justifications.

C.3. Better than standard semantics?

Proofs: finitely generated from finitely many rules and axioms; proof-correctness is linear.

Even if not all axioms are known,
correct proofs from the known axioms exist.

Axioms can be learnt
from interactions between proofs, cf. infra.

Includes argumentative aspects of semantics.

C.4. Limitations

Not all axioms are not known.

We do not include justification of why saying rather than not saying, lying etc.
A justification for saying That's not a big deal. *might be that the speaker wants to minimise a mistake, although it actually is big deal.*

Negation is an obstacle to compositionality:
At most one of A and $\neg A$ is provable.
At least one of $[\![A]\!]$ or $[\![\neg A]\!]$ is empty.

C.5. Proofs and refutations

Problem with negation
$\rightarrow$ proofs and refutations on a par ?

Pseudo proofs like in Ludics
(daimon, circular proofs,...)

Interaction between proofs and refutations:
proof normalisation reveals axioms.

C.6. Justifications as informal proofs

Justifications/proofs in natural language?
Need for an unambiguous language.

Mathematical practice. Natural language (especially for reasoning rules) with some computations and formulas when needed.

Natural logic: Aristotle syllogisms today !
Sentences with several quantifiers, numbers using fixed grammatical patterns.
For simple maths and every day reasoning.

D Conclusion

Formal and informal **proofs** as meanings for *mathematical or natural language* assertions.

Natural outcomes:

- **argumentative** aspects of meaning.
- **coherence** of discourse and dialogue (proving there is no model is difficult)

D.1. Maths?

Analysis mathematical practice,
teaching, didactics

Type Theory, Topological Models,...
Negation? Identity? Quotient?

The chair did not bark.
Rex is not a dog?
I read the same book.
I did not read this book.