Exercice 1. A mathematician has chosen in $\{0,1\}^{10}$ a subset of k binary words $x_1, x_2, \ldots, x_k$ so that the Hamming distance between any two of these words is $\text{dist}_H(x_i, x_i) \geq 6$. Prove that $k < 20$.

Exercice 2. The parity check matrix of the Hamming code with codewords of length 7 is

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

(a) How many errors does this code correct ?

(b) Does there exist a binary word x of length 7 that is not a codeword for this code and that cannot be obtained from any codeword by inverting one bit ?

(c) Is the bit sequence $x = (0010100)$ a codeword of this code ? If not, which bit should be inverted to obtain a codeword ?

(d) Is the bit sequence $x = (1111111)$ a codeword of this code ? If not, which bit should be inverted to obtain a codeword ?