UM 2025. Cours HAI906I **«Calcul formel avancé et applications»**, Master 2.
Exercises complementing lectures 1–3.

Exercice 1. You are given a balance scale without any weights, which allows you to compare the weights of any two stones. How many weighings are required to guarantee sorting n stones by weight ?

(a) $n = 3$.
(b) $n = 4$.
(c) $n = 5$.

Exercice 2. Let (X, Y, Z) be a triple of jointly distributed random variables, each taking values in a finite set. Prove that

$$H(X, Y, Z) \leq H(X, Y) + H(X, Z) + H(Y, Z).$$

Exercice 3. (a) Suppose a certain binary code consists of codewords of length 5 and can correct 1 error. Can this code contain 6 codewords ?

(b) Suppose a certain binary code consists of codewords of length 7 and can correct 3 errors. How many codewords can such a code contain ?

Exercice 4. Prove that there exist a real number $c > 0$ and an integer n_0 such that for all $n > n_0$ there exists a binary code

$$\{c_1, c_2, \ldots, c_N\} \subset \{0, 1\}^n,$$

containing more than 2^{cn} codewords and capable of correcting at least 1% of errors.

Exercice 5. In the game *Guess the Number*, the first player thinks of an integer between 1 and 15, and the second player asks yes/no questions. One of the first player's answers may be false. How many questions are required to determine the chosen number ?