Random number generator: testing and whitening

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invitation to an internship (2020)

What is a random sequence of digits?



- 0

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Can you guess which sequence here is truly random?

- 110010010000111111011010100001000001
 (btw, the last sequence is a binary expansion of *π*)

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In some sense, all 40-bit sequences are equally random (they all have the same probability 2^{-40} in a fair coin model), but some look more random than others.

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$$x_1...x_n$$
 is random means $C(x_1...x_n) \approx n$
 $x_1...x_n$ is non-random means $C(x_1...x_n) \ll n$

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Big practical question: where do we get truly random bits?

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2		TABLE OF RA	DOM DIGITS	
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Rand Corporation, A Million Random Digits with 100,000 Normal Deviates (1955)

26269

20971 87749

66281 31003

[random digits kindly generated for us in 1955]

62290 64464

90429 12272

00682 27398

27124 67018

95375 05871

20714 53295

82760

41361

93823 43178

07706 17813

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00052

00053

00054

73189 50207

75768 76490

54016 44056



[random digits from a noise in an electric circuit]



[random bits from quantum phenomena]

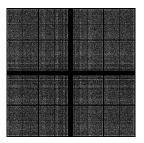
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100 kHz



default rate 2.5 MHz

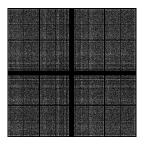
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5 MHz

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			200 003 002 003
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5 MHz

successful whitening: XOR of 3 (apparently biased) data flows looks pretty random

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You cannot trust blindly the standard implementations of randomness tests (mathematically unsound tests, errors in the code).

The challenges:



cleanup/enhance existing randomness tests



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- apply (certainly non-perfect) pseudo-random generators to produce useful random objects (e.g., error correcting codes)

Internship proposal:

Generation of random bits is a classical problem known in the context of pseudo-random generators and also in connection with of truly random physical processes (there exist electronic devices that produce random bits using an unpredictable physical noise or intrinsically nondeterministic quantum phenomena). However, the quality of physical generators of random bits remains badly founded and poorly tested. The first objective of this project is an experimental study of the validity and quality of several physical random numbers generators.

When we talk about the quality of random or pseudo-random generators, we have to use randomness tests. The second objective of the project is an inventory and revision of statistical tests for random and pseudo-random generators. We suggest to improve the quality of statistical tests and develop new techniques of "whitening" that improves the quality of non-ideal sources of random bits. Another axis of the project is a conversion of various probabilistic proofs into unconventional randomness tests.

Prerequisites: Basic knowledge of probability and statistics, and solid programming skills. The main tools in the project are pretty standard: C / gcc / Linux. The project requires not only writing your own code but also reading and maintaining the code that already exists.