Proposition de stage M2

Titre: *Modification problems on graphs: algorithms, logic, and combinatorics.*

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Description du sujet (en anglais)

Keywords: Parameterized Algorithms, Graph modification problems, Algorithmic meta-theorems, Logic, Graph Minors, Tree Decompositions.

Parameterized complexity proposes a refined analysis of computational problems, focusing on structural aspects that may drastically reduce the exponential burden of the corresponding algorithmic solutions. It was introduced in the 90's by Downey and Fellows and currently constitutes a fully developed discipline of Theoretical Computer Science (see [3] for a recent textbook). The idea is that, beyond the overall input size, key secondary parameters fundamentally affect the computational complexity of problems and reveal opportunities for designing algorithms that can still be considered efficient under a more loose, but still legitimate, concept of efficiency. In parameterized problems, apart from the input size, a second variable is a *parameter* that measures certain structural particularities, preferably those that are common in real applications. This approach yields algorithms with running time that is exponential in the parameter (denoted by k) but polynomial in the input size (denoted by n). In technical terms, we are looking for algorithms that run in time $f(k) \cdot n^c$, where f is a function depending only on the parameter and where c is a constant. These algorithms are called *Fixed Parameter Tractable* algorithms, or simply FPT-algorithms.

Algorithmic meta-theorems. An active line of research in Logic and Algorithms is the quest for *algorithmic meta-theorems* (abbreviated as AMTs). According to Grohe and Kreutzer [10], algorithmic meta-theorems state that under specific conditions, the automatic derivation of efficient algorithms is possible for wide families of algorithmic problems. These conditions are typically linked to the descriptive complexity of the problems (via Logic) and the structural properties of their inputs (via Combinatorics). Algorithmic meta-theorems play an important role in the theory of algorithms as they reveal deep interplays between Algorithms, Logic, and Combinatorics. Not surprisingly, most AMTs that have appeared so far use the multivariate framework of *parameterized algorithms*, and this is because the concept of problem parameterization (i.e., the choice of the parameter k) responds directly to the demand for quantifying the structural characteristics of problem inputs.

Graph modification problems. A *graph modification problem* asks whether it is possible to apply a series of modifications to a graph in order to transform it to a graph with some desired target property. Such problems have been the driving force of Parameterized Complexity where parameterization quantifies the concept of "distance from triviality" [11] and measures the amount of the applied modification. Classically, modification operations may be vertex/edge deletions, edge additions/contractions, combinations of them, or other (local) modification operations. In their generality, such problems are

NP-complete and much research in Parameterized Complexity is on the design of FPT-algorithms for parameterizations defined by different measures of the modification operation. The target property may express desired structural properties that respond to certain algorithmic or combinatorial demands. A widely studied family of target properties are minor-closed graph classes [14, 15], however, other families of target properties have also been considered, such as excluding some fixed graph under certain partial ordering relations [6, 8, 9]. More involved modification measures of vertex set removals, related to width parameters, have been considered very recently [1, 2, 5, 13].

Recently, in [7], a *compound logic* was proposed, combining two types of sentences, expressing graph modification: the *modulator sentence*, defining some property of the modified part of the graph, and the *target sentence*, defining some property of the graph obtained after the modification. In this framework, modulator sentences are expressible in Monadic Second-Order Logic (MSOL) and have bounded tree decomposability, while target sentences combine First-Order Logic (FOL) and minor-exclusion. According to the meta-algorithmic result of [7], for this compound logic, model-checking can be done by an FPT-algorithm running in time $f(\theta) \cdot n^2$, where f is some constructive function depending on the sentence θ defining the problem. This permitted the automatic derivation of FPT-algorithms for a wide family of modification problems on graphs and the unification of several previous results to a single theory.

Topic of this internship. This stage proposal aims at investigating the parameterized complexity of modification problems that go beyond the current state of the art on algorithmic design for modification problems. This requires dealing with problems (or families of problems) that go further than the meta-algorithmic horizon drawn in [7]. For this, we are interested in the algorithmic study of

Modification problems for target properties that are not expressible in FOL.

Such a project asks either for the extension of the known algorithmic techniques or the invention of new ones. It also requires the investigation of the inherent tractability limitations of graph modification and prove complexity lower bounds delimiting their tractability horizon. The goal of this internship is to make some first steps in this direction. More precisely, two avenues that will be considered are the following:

- Study graph modification problems where the target property is bipartiteness, expressed by a sentence φ (notice that bipartiteness is not a FOL-definable property, therefore φ cannot be a FOL-sentence). Fiorini, Hardy, Reed, and Vetta [6] designed an FPT-algorithm checking whether a graph can be made planar and bipartite by removing k vertices. Can we extend this result by additionally demanding any FOL property? Instead of demanding planarity, can we ask for some other property such as containment in some minor-closed graph class? There are insights suggesting that such questions can be positively addressed.
- Bipartiteness is certainly a first non FOL-definable target property to consider. However, it is possible to fit this in a more wide family of target properties. Such a general framework is offered by the concept of *odd-minor exclusion* which has not been, so far, investigated from the point of view of modification problems. We plan to consider known structural theorems on odd minors [4, 12] and investigate up to which extend they may be combined with the techniques of [7] so to yield FPT-algorithms for graph modification problems. Apart from odd-minors, we also plan to consider other known graph structural theorems and examine whether they can be used (or adapted) so to provide algorithmic (or meta-algorithmic) results for general families of graph modification problems.

Research enviroment: The AlGCo team has a strong background on the proposed research. AlGCo is one of the leading research teams in France in Parameterized Computation, Structural Graph Theory, and its links with diverse fields of algorithms, including the interplay between Logic and Algorithms. Members of the team have contributed to international conferences and participated in research projects that are directly or indirectly related to meta-algorithmic techniques and have an increasing interest in the interleave between Logic and Combinatorics. The publication record of the team of the last 5 years can be accessed at *http://www2.lirmm.fr/algco/publications.php*.

Candidate profile: The candidate should have a solid background in Graph Theory, Algorithms, Complexity, and Logic, as well a strong motivation to work on related areas and principally in Parameterized Complexity. Publications and research experience in related fields will be appreciated. A proficient level of spoken and written English is mandatory.

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