# SEGMENTATION OF CROSS-SECTIONAL IMAGES USING FUZZY LOGIC 

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Abstract - In this paper, we present a new segmentation method for range images consisting of a set of planar cross-sectional contours. Our approach is novel in that it uses fuzzy criteria for grouping primitives and identifying homogeneous regions. We have tried our method with images provided by a structured light sensor. In this case, the image sequence corresponds to the scene profiles obtained with the successive positions of a rotating plane of laser light. We assume that the object surfaces can be modelled by a set of quadratic patches. The primitives used for region segmentation result from the approximation of the light profile by second order curves. An efficient tracking of these noisy curves is achieved by using a fuzzy decision-making algorithm. Region growing is then performed by our method by matching 2D curves from the image sequence.

We present results obtained with real scenes consisting of multiple objects of arbitrary shapes. They show that an efficient surface segmentation may be obtained with fewconstrained environments including planar or curved shapes.

## 1-INTRODUCTION

With many 3D sensing systems, the surface of the viewed scene is defined by a set of planar cross-sectional contours. In manufacturing applications, active sensing techniques using, for example, a scanning process provide raw data which are displayed on a regular array. For instance:

- With time of flight measurement systems (lasers, ultrasound sensors, ...), the scanning axes are generally two perpendicular rotation axes.
- In the case of structured light sensors, the illumination pattern has a simple and regular geometry. It consists of a stripe, a multi-stripe, or a grid of points or of lines, etc.
The purpose of the present work can be formulated as follows: let us consider a scene of 3D objects intersected by a set of neighbouring planes obtained by successive rotations or translations. The 3D points measured on the objects belong to the intersections of their surfaces with the planes. The problem that we want to solve is: how to analyze the consecutive scene profiles in order to segment the shape image into homogeneous regions?

Classical segmentation methods which are applied to range images can be divided into two groups: region based methods and boundary based methods. Region based methods look for topological [1] or geometrical [2] similarities to group 3D points or elementary patches into surface regions. For instance, normals to a surface may be considered as a common local feature. Partitioning them involves thresholding using a histogram analysis [3]. Boundary based methods try to find significant changes that separate regions by isolating discontinuities in both depth and surface orientation. Differential geometry is often used to find region boundaries [4]. All these segmentation techniques use dense 3D data and don't take advantage of the data spatial organization which is encountered in cross-sectional images.

In this work 3D sensing is performed via structured light images. However, we show that an efficient 3D shape segmentation can be done without computing any 3D surface points. Our method is based on the direct analysis of the parameters of the projected stripes in the image frame and of their connectivity relations. We show how it is possible to track stripe parts in the consecutive images and to match them in order to create regions.

Our method solves the surface identification problem by using a very general constraint that holds in environments consisting of a jumble of manufactured objects. We suppose that surfaces are "regular" enough and that they can be locally approximated by second order patches. We make the assumption that these patches are larger than the stripe spacing given by the scanning system.

This paper includes three major sections. In section 2, we present the problem we want to solve and we briefly describe our application and the algorithms developed for the preliminary data processing. Section 3 shows how to achieve the tracking of profile parts in order to segment the topographic image by using a fuzzy aggregation of criteria. Section 4 presents our conclusion as well as experimental results.

## 2 - PROBLEM STATEMENT



- Fig. 1 : The sensor-


## 2.1 - THE SENSOR

Our vision sensor consists of a CCD camera and an optical projection system that generates a plane of red light from a HeNe laser (Fig. 1). The projected light is scanned across the scene by means of a rotating mirror controlled by a galvanometric device. At each scanning step, we obtain a grey-level image showing points on the scene which are illuminated by the light plane (Fig. 2a).

In most of applications, the variable range between sensor and objects causes defocussed images for the projected stripe. The first processing step consists of extracting the scene profile, i. e. the skeleton of the projected pattern which represents the intersection of a perfect plane with the 3D objects (Fig. 2b). The method we use is based on statistical properties of the signal we wish to find. The reader is directed to [5] and [6] for a detailed explanation of the algorithms developed for the skeleton extraction.

At the end of this process, a set of one pixel wide continuous segments is obtained on the image. Localization of any point of each segment is subject to a


- Fig. 2a : The original image of a laser stripe -
measurement of variance. In the example shown on Fig. 2b, four stripe parts have been identified.


## 2.2 - PROFILE SEGMENTATION

Our aim is to approximate object surfaces by quadratic models. It can be proven that the intersection of a quadric by a plane is a conical curve [7] whose homographic projection is also conic. Consequently, we have developed a segmentation method that allows us to approximate the laser stripe by a set of adjacent second order curves [5]. We present here a brief description of this algorithm which locates the skeleton discontinuities such as breaking, angular, retrogression or bending points. A detailed presentation of the method has been published in a previous paper [8].

Let $\mathrm{x}=\mathcal{L}(\mathrm{y})$ be the noisy curve resulting from the previous processing (Fig. 2b). Discontinuities on this curve correspond to maxima or zero crossing of the second derivative of $\mathcal{L}(y)$. Classical differential operators being sensitive to noise, we compute $L^{\prime \prime}$, which is a good estimate of $\mathcal{L}^{n}$, the second derivative, by using a symmetric exponential filter [10] which preserves the accuracy of the discontinuities location [9]. This derivative includes a residual noise which is evaluated.

The analysis of the smoothed $\mathrm{L}^{\prime \prime}$ signal consists of locating both sign changes and local maxima, in order to subdivide $\mathcal{L}(y)$ into segments that may be approximated by second-order curves. Nevertheless, the stripe segmentation must take into account the estimated noise for each analyzed point. For instance, let us consider the examples of second derivatives obtained with a vertical and an oblique stripe (Fig.3). We can see that many non significant zero crossings and extrema can be found.


- Fig. 2b: The laser stripe skeleton -


Fig. 3 : Second derivatives for a vertical stripe (a) and for an oblique stripe (b)
Consequently, for identifying singularities, we consider that:

- a bending point is a point for which $\mathrm{L}^{\prime \prime}$ is small, compared to those of the set of its neighbouring points;
- a breaking point is a point for which $\mathrm{L}^{\prime \prime}$ is large, compared to those of the set of its neighbouring points.
In order to locate the singularities, taking the noise into consideration, we associate a fuzzy measure of straightness to each point. The straightness at a point y is considered as the degree of belonging, $d(y)$, of this point to the fuzzy set $\mathcal{A}$ of the points which have a small second derivative. This fact of having a small second derivative is referred to a measurement of variance evaluated at each point by a specific process. More detail can be found in [6].

Let $\sigma^{2}(y)$ be the variance of $L^{\prime \prime}(y)$ at $x=L(y)$, the fuzzy straightness at ( $\mathrm{x}, \mathrm{y}$ ) is written as:

$$
d(y)=\exp \left(-\frac{\left(L^{\prime \prime}(y)\right)^{2}}{\sigma^{2}}\right)
$$

Moreover, the global fuzzy straightness of a segment $S$ is defined as being the generalized relative Hamming distance [13] between $\mathcal{A}$ et $S$. At the $\mathrm{i}^{\text {th }}$ iteration, we have:

$$
s_{i}=\frac{i-1}{i} \cdot s_{i-1}+\frac{1}{i} \cdot d\left(y_{i}\right)
$$

When $s$ is not consistent with $d$ at a specific point, the system searches for a singularity:

- A maximum of $L^{\prime \prime}$ if $s<0.5$ and $d>0.5$,
- A minimum of $L^{\prime \prime}$ if $s>0.5$ and $d<0.5$.

At the end of this process, the skeleton of the laser stripe consists of a set of 2D primitives called fragments which can be approximated by second order curves.

## 3 - TRACKING WITH LIKENESS

This section is an attempt to show that there is an alternative for token tracking in an image sequence. Instead of either logical matching or statistical inference, a fuzzy relation can be evaluated. This relation is known as likeness.

Why use a new theory for tracking? Statistical approaches to feature tracking in a dynamic image sequence have been widely adopted by many authors [10][11]. Presently, most of these methods aim at tracking straight line segments extracted from the detected edges. These segments are represented by their geometric and dynamic parameters which are updated by a Kalman filter, before matching with the observed tokens. However, these algorithms are not adapted for tracking any kind of noisy curves, and they cannot take into consideration a possible deformation of the curve from one image to the next.

In classical algorithms, matching and tracking are based on similitude properties. We propose the use of a more general concept: the likeness. These two concepts differ in the fact that similitude is a transitive property, while likeness is not. Likeness can be described by a fuzzy variable.

Our matching process uses a set of redundant criteria to estimate the likeness between each fragment $F_{1}$ of the previous image $I_{1}$ and each fragment $F_{2}$ of the current image $I_{2}$.

Indeed, using classical tracking methods will presume that the observed features satisfy some statistical assumptions such as probability distributions. For deformable and noisy curves, one could hardly expect the distribution of the patterns to correspond precisely to the presumed distribution. For these reasons, in a general case, feature tracking must be considered essentially fuzzy. Fuzzy set concepts proposed by Zadeh [12] have already been applied to pattern recognition.

However, since vision systems provide noisy and incomplete information, we have chosen to use information redundancy and to match tokens with different kinds of criteria. According to the properties of this information, these criteria may be logical, statistical, heuristic, or fuzzy.

For a logical decision, one must associate a logical variable to any criterion involved in the decision ; the same statement holds for a probabilist decision and a probabilist variable. This association variable/criterion is clone via a test, by a specific function called a predicate.

Similar assertions and definitions can be used in the case of fuzzy logic: a fuzzy variable must be associated to any criterion. This association is done by a fuzzy predicate, which acts upon a test. If this test is fuzzy or logical, the predicate is nothing but an identity function. Therefore, we are only interested in statistical or heuristic tests.

We now present the method we use for evaluating the likeness of two geometric primitives.

Kaufmann [13] presented likeness as a fuzzy symmetric and reflexive relationship. Likeness may also be seen as a weak similitude which worsens as the tokens are matched. More precisely, likeness between two geometric features is an aggregation of several criteria, leading to satisfaction of a common aim [14].

A likeness evaluation between two features is performed in four steps:

1) Modeling the feature,
2) Determining the criteria,
3) Using what we called fuzzy predicates to join a fuzzy value with each criterion,
4) Linking those fuzzy values by means of a fuzzy logic algorithm.

## 3.1 - MODELING

The problem is to choose an appropriate representation which is characteristic of the geometric properties of the curve fragments (which approximate the laser stripe), since their tracking will be based on this representation. For that purpose, five vectors of static parameters and a measurement of displacement are used.

Each primitive can be approximated by a second order function. Moreover, due to the sensor geometry, the curve has only one intersection point with each image line. Consequently, we have shown that a simple parabolic model is sufficient to describe this curve. This model can be written:

$$
x=a y^{2}+b y+c
$$

The identification of the $a, b$ and $c$ parameters is performed by a least square algorithm weighted by mean of the location variance at each point. This method allow us to estimate the variance/covariance matrix of the parameter vector $\left|\begin{array}{l}a \\ b \\ c\end{array}\right|$ by back propagation of measurement errors [15]. The others static parameters are given by the preprocessing:

- the angular position of the laser plane which has been used for acquiring this image,
- a list of points with their coordinates and their location uncertainty,
- the two end-points with their coordinates ( $x, y$ ) characterized by $d(x, y)$, the local straightness. In fact, for a singular point, $d(x, y)$ is near 0 if it is an angular point and near 1 if it is a bending point.
- The global straightness $s(\mathrm{~F})$ of a given fragment F ; $s(\mathrm{~F})$ is near 1 for a regular straight fragment, otherwise it is near 0 .
In order to improve the fragment matching, an evaluation of $D$, the average spacing between two fragments belonging to the same surface patch as well as an estimation of the variance of $D$ are added. $D$ and the variance are updated by mean of Kalman filter.


## 3.2 - CRITERIA

In this section we describe the criteria that are used to evaluate the likeness between two fragments. Each of these criteria expresses the homology of a specific geometric property. They have to be robust enough globally to palliate three major problems:

- Data issued from the preprocessing are noisy (due to the image sampling and to the skeleton extraction).
- We can assign only an approximate model to each primitive.
- The shape of the viewed surfaces being not regular enough, some small deformations of the curves may appear during the tracking.
Our tracking approach is based on the assumption $\mathcal{H}$ defined as follows:
$\mathcal{H}$ : the angular deflection is sufficiently small in order to allow us to presume that the surface patches are regular between two positions of the light plane.
Let $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ be two fragments and $\mathcal{R}\left(\mathrm{F}_{1}, \mathrm{~F}_{2}\right)$ the likeness we want to evaluate. Let $F_{1}^{\prime}$ be the fragment $F_{1}$ displaced by $D$. Within the terms of the previous assumption $\mathcal{H}$, the following criteria are considered:
- The Bhattacharyya distance [16] between the parabolic approximations of $F_{1}^{\prime}$ and $F_{2}$,
- The Mahalanobis distance which characterizes the belonging of some points of $\mathrm{F}_{1}^{\prime}$ to the parabolic approximation of $\mathrm{F}_{2}$ and vice versa,
- the fuzzy distance between the straightness and the fuzzy disparity of the two end point types,
- the heuristic overlapping of the projection of $F_{1}^{\prime}$ and $F_{2}$ on the vertical axis.
We take into account the initialization step by using a logical criterion. It allows us to inhibit the effect of some other criteria, such as that of Battacharyya distance during the initialization.


## 3.3 - PREDICATES

A logic predicate is an association between a logic value and any other value (real, symbolic, etc ). Similarly, a fuzzy predicate is a function associating a criterion to a fuzzy variable. Fuzzy predicate is an evaluation of the agreement of a specific criterion to a given matching.

In this case, only the statistic (statistic distance) and heuristic criteria (overlapping) need to use such functions.

Notation: $6=\mathcal{P}_{(\mathrm{B})}$ is the fuzzy value associated with B by the predicate $\mathcal{P}$.

### 3.3.1 - PREDICATES ON STATISTICAL CRITERIA

Bhattacharyya distance, as Mahalanobis distance, may follow a $\chi^{2}$ law [17]. The use of a $\chi^{2}$ test is the same as finding a real segment $\left[\mathrm{B}_{1}, \mathrm{~B}_{2}\right]$ corresponding to the limits of the $99 \%$ confidence interval of matching hypothesis acceptance. Logic predicate indicates the belonging of B , the Bhattacharyya distance, to this segment. In order to evaluate a fuzzy predicate, this segment is considered as the $\alpha$-cut at 0.5 [13] of the fuzzy set which corresponds to the matching hypothesis acceptance (see Fig. 4).


- Fig. 4: First predicate on Bhattacharyya distance -

In order not to reject exact coincidence (most unlikely but desirable) a second predicate is added that correspond to a great likeness. The maximum limit $\mathrm{B}_{3}$ of this new segment corresponds to the maximum limit of the $50 \%$ confidence interval (see Fig. 5).

-Fig.5: Second predicate on Bhattacharyya distance -

Similar predicates are associated to the Mahalanobis distances. After all, a second evaluation of the straightness is obtained by comparing between the coefficient of the highest power term of the parabolic approximation and its variance. This comparison is done via a fuzzy predicate acting on a Student test [6].

### 3.3.2 - PREDICATES ON HEURISTIC CRITERIA

This criterion concern the overlapping of the two projections of $F_{1}$ and $F_{2}$ on the $x$ axis, with respect to the predicted displacement D (see Fig. 6).


Fig. 6: Fragments overlapping
Two redundant predicates, $o_{1}$ and $o_{2}$, are established for this criterion. They can be respectively assimilated to a majority vote and to a proportional vote:

$$
o_{1}=\frac{S_{1} \cap S_{2}}{S_{1} \cup S_{2}} \quad o_{2}=\frac{S_{1} \cap S_{2}}{\operatorname{MNN}\left(S_{1}, S_{2}\right)}
$$

### 3.3.3 - PREDICATES ON LOGICAL AND FUZZY CRITERIA

For these criteria, the predicate is the identity function.
We take into account the initialization step by using a logical criterion. It allows us to inhibit the effect of some fuzzy variables in the decision process (such as Battacharyya distance).

We also use the fuzzy criteria which are included in the primitives model: the end points type $\mathbb{d}$ and the fuzzy straightness $s$ of the primitives (see section 2.2).

## 3.4 - AGGREGATION OF THE CRITERIA

In order to obtain the fuzzy likeness relation, classical fuzzy logic laws are used. Three sets of laws are characterized which are objective, subjective and equivocal laws.

Laws that use MIN and MAX operators are called objective laws. They are used to associate one criterion to another, or more precisely, to associate the predicates of these criteria, of which no a priori correlation is known. For example, the fact that the end points of two fragments may be of the same type may not seem a priori related to the fact that their parabolic approximation be close.

The subjective laws are fuzzy functions which behave as boolean functions when given boolean arguments. These laws may be divided into so called optimistic and pessimistic laws. Given two arguments, a pessimistic law is one that give a result that is worse than the equivalent objective association of these arguments (for example $\mathrm{a} . \mathrm{b} \leq \operatorname{MIN}(\mathrm{a}, \mathrm{b})$ ). It expresses an a priori conjunction of the corresponding criteria. On the other hand, an optimistic law gives a result that is better than the equivalent objective association (for example $\mathrm{a} \hat{\mathrm{t}} \mathrm{b}=\mathrm{a}+\mathrm{b}-\mathrm{a} \cdot \mathrm{b} \geq \operatorname{MAX}(\mathrm{a}, \mathrm{b})$ ). It expresses an $a$ priori disjunction of the two events. Thus the association of these two fuzzy variables measuring the straightness of a fragment is of a pessimistic type, since one has more confidence in the straightness of a fragment if both measurement are true.

Equivocal laws show an intermediate behaviour which may not be linked to a classical logic law. The best example of such a law is the mean function, which allows among other things to express a fuzzy "if-then-else" statement. If $\mathbf{c}$ then a else $\mathbf{b}$ is written as: $\mathbf{c} \cdot \mathbf{a}+(\mathbf{1}-\mathbf{c}) . \mathbf{b}$.

We present here an example of fuzzy logic rule which is used for obtaining the likeness of two fragments.

— Fig. 7a -

The Battacharyya distance is not a reliable data when the assumption of a curve line is made while two straight lines are compared. This is due to numerical reasons. That is the reason for which a second distance is estimated with assertion of two straight lines (in the presence of ambiguity). The computation of the predicate 6 on this distance is achieved by the rule: «if $F_{1}$ and $F_{2}$ are straight lines, then the question is "is the distance $\mathcal{B}$ between $F_{1}$ and $F_{2}$ small, assuming they are straight?", else same question assuming they are curved».

Let the smallness of $\mathcal{B}$ be $\sigma_{d}$ assuming we compare straight lines and $\sigma_{c}$ without this assumption. Let $\alpha_{1}$ be the straightness of $F_{1}, d_{2}$ the straightness of $F_{2}$, then the likeness $\sigma$ will be:

$$
\sigma=\left(\mathrm{dr}_{1} \wedge \mathrm{dr}_{2}\right) \cdot \sigma_{d}+\left(1-\left(\mathrm{dr}_{1} \wedge \mathrm{dr}_{2}\right)\right) \cdot \sigma_{c} .
$$

where $\wedge$ is the objective AND.

## 4 - EXPERIMENTAL RESULTS

The algorithms we propose have been implemented on a PC computer and tested on indoor scenes including planar and curved objects. Here, we report some results obtained with manufactured objects, where the striped images are superimposed on the images obtained with the ambient lighting. On Fig. 7a, all the stripes obtained with 50 angular positions of the light plane are shown. Fig. 7b and 7c present extraction of planar surfaces. On Fig. 7d and 7e, the cylinders are clear. It highlights the ability of the fragmentation process to detect retrogression points. Finally, the last figures show the robustness and the adaptability of the algorithm in case of inclined curve surface (Fig 7f) and irregular planes inclined (Fig.7g) or not (Fig. 7h).

— Fig. 7b -

— Fig. 7c -

— Fig. 7e -

— Fig. 7g -

— Fig. 7d -

— Fig. 7f -


- Fig. 7h -

Some Examples

One of the major limitations of our algorithm is the computation time needed by the image preprocessing (skeleton extraction) which is about two thirds of the computation time. Presently, all of our programs are developed in C language, on a PC computer ( 30286 ). Because of the lack of memory which involves many data exchanges with the disk storage unit, this program takes very long to run on such machine. The execution time depends on the number of the analyzed stripes in the image. For one stripe, it increases with the size and the number of shadow zones along this stripe, and with the number of extracted primitives. For instance, we have estimated that the processing of one stripe on our PC takes between 40 s and 1 mn .

We are currently studying the possibility of using a dedicated hardware to realize the skeletonizing. The first estimation of the computation time is about 80 ms to obtain the skeleton of a $512 \times 512$ image whatever its complexity is. The same skeletonizing algorithm has been implemented on a SUN station (SPARC II). We have obtained computation times varying between 0.6 and 0.8 s depending upon images complexity.

These first results are promising because then the total processing times for a depth image composed of 50 stripes will not exceed 20 s . This mean a reduction of the computation time in a ratio of 150 to 1 .

## 5. CONCLUSION

We have presented algorithms for solving the segmentation problem of cross-sectional images obtained with a structured light sensor. The approach is new, since:

- The data which are used are not 3D points, but only the 2D points extracted from the image of the crosssectional contours.
- The method takes advantage of the spatial organization of the data which is encountered in cross-sectional images. Indeed, it uses primitives which are curves extracted from the stripe profile.
- The tracking of the primitives used for region growing is achieved by a fuzzy decision-making algorithm which leads to reliable and accurate results.
The efficiency of this segmentation method is due to the fact that during the different steps, it takes into account the data noise and the models uncertainty.

Future extensions of this work are essentially directed toward the application of fuzzy tracking methods for analyzing range images used for solving navigation problems for mobile robots.

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