MULTIBEAM SONAR IMAGE MATCHING FOR TERRAIN-BASED UNDERWATER NAVIGATION

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ABSTRACT

This paper presents a terrain-referenced method for positioning an underwater vehicle equipped with a multibeam sonar. The local bathymetric profiles provided by this sensor are matched with a digital elevation map (DEM). The absolute position and heading of the vehicle with respect to this reference map are estimated by correlating a part of this map with the on line altitude measurements. We propose an extended approach to the classical matching methods that takes into account the non regular sampling of sonar data in the correlation space. This technique has been successfully applied to real terrain data obtained from the Var underwater canyon (France).

I. INTRODUCTION

A major problem in autonomous navigation of an underwater vehicle is to continually estimate its localization with respect to the environment. Different methods can be used to reduce the positional uncertainty that results from the accumulation of dead-reckoning errors.

For instance, transponders can be installed on the seabed (long baseline system) and the vehicle position is estimated via a triangulation algorithm. Such a solution provides localization only in a limited area. In short base line systems, the sensor is fitted to a surface ship, which must follow the underwater vehicle in order to remain above it. It appears that these positioning methods are not intended for long distance navigation purposes. Position measurements with respect to natural seamarks seem to be a more appropriate solution.

Stereovision [1,2,3] or dynamic vision based on the analysis of monocular video images sequences are a very active field of search for applications to ground vehicle localization. Different solutions using monocular vision for navigation have been studied such as: optical flow, template matching, feature correspondence. Such vision techniques can be used successfully in underwater environments for operations involving short-range observations (ROV stabilization for maintenance tasks [4], inspection of bottom cables or pipelines, seafloor observation). We are interested in terrain-based navigation, which involves to observe the seafloor relief with altitude values ranging from one hundred to several hundred meters. Video cameras are not suited to this kind of application because of the lack of visibility in the underwater environment.

Sonars provide sensing capabilities at greater ranges than those offered by video means. In the last three decades, significant results have been obtained with mapping techniques applied to seafloor inspection, cartography, object detection and classification [5], ... Some recent works concern terrain-based navigation [6,7,8]. In [7,8], the data processing requires a regular map of the observed area, i. e. a 2D array of uniformly spaced altitude values. Such a local image is built by registering consecutive bathymetric profiles, and by extrapolating this original data set in order to obtain a regular sampling. Generally, solutions involving matching or features correspondence require an image preprocessing (filtering, segmentation, critical points extraction,...) which is time consuming. The most efficient matching algorithms make use of attributes, which are invariant under size, orientation and displacement.

In order to obtain a real time localization system, we have developed a matching procedure, which differs from previous solutions in several aspects. Firstly, it uses 1D local measurements, which are simple bathymetric profiles, instead of 2D local maps, which require the grouping of consecutive profiles. Secondly, rough altitude data are correlated with the reference map without any preprocessing. In addition, a tracking process based on Kalman filtering allows to reduce the search area during each new localization step.

This paper is organized as follows. In section II, the sonar images and the notations used for the algorithm presentation are briefly described. The theoretical part of our work is presented in section III. We propose an original solution for the matching of 1D non-regular data with a 2D reference map where data are uniformly sampled. Two approaches are described: a statistical approach and a fuzzy approach. In section IV, we give a brief a overview of the tracking algorithm used to reduce the size of the correlation space. The experimental results presented in section V have been obtained with a reference map built with real terrain data from the Var underwater canyon (France).

II. SONAR IMAGES

In our algorithm, we consider that the vehicle carrying the sonar follows a trajectory with a variable depth P. The multibeam imaging and profiling sonar acquires range data by successive cross sections. For each measurement profile, the sensor emits 59 acoustic beams (k = 1,..., 59) separated by 1.5 degree, in a plane perpendicular to the track followed by the vehicle. Figure 1 shows the acquisition geometry and specifies the different variables and frames, which will be used in the next sections. The altitude Z_k of a point P_k is a function of the range measurement ρ_k, and of the angle θ_k of the beam k with a vertical axis.

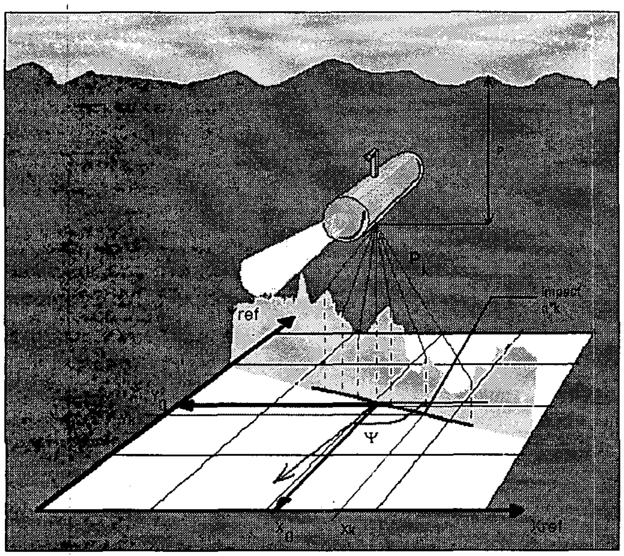

Figure 1. Data acquisition.

(X_0, Y_0) are the position parameters of the sensor and Ψ its heading in the frame ($Xref$, $Yref$) attached to the reference map.

III. NON REGULAR PATTERN MATCHING

A. EXPECTED VALUE OF A MATCHING METRIC

Let us go back to the definition of expected value of a matching metric. Let u and v be two signals to be compared and a matching metric. Expected value of (u,v) is:

$$E(\partial(u,v)) = \sum_{i=1}^{Min(I,J)} \partial(u_i, v_i) \tag{1}$$

where u_i and v_j are discrete values of u and v, with (i=1...I) and (j=1...J).

With a non-regular sampling of u and u, equation (1) may be generalized as:

$$E(\partial(u,v)) = \sum_{i=1}^{I} \sum_{j=1}^{J} \partial(u_i, v_j) \cdot p_{ij} \tag{2}$$

where p_{ij} is the joint probability:

$$p_{ij} = Pr(u_i = v_j)$$

If x and y are identically sampled (for instance, when matching two video images with an integer shift), p_{ij} is:

$$\begin{cases} p_{ij} = 1 & \text{if } i = j \\ p_{ij} = 0 & \text{if } i \neq j \end{cases}$$

which corresponds to the previous equation (1).

In the context of this paper, data to be matched are respectively the depth Z_k of each impact k of the sonar scan (k=1,..., K) for a given position of the submarine and T_{ij} the depth of the cells (x_i, y_j) of the DEM.

Generalizing equation (2), the expected value of the matching metric between T_{ij} and Z_k becomes:

$$E(\partial(T,Z)) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \partial(T_{ij}, Z_k) \cdot Pr(x_i = x_k, y_j = y_k) \tag{3}$$

where (x_k, y_k) are the estimated coordinates of point P_k.

We can use different criteria to determine the degree of resemblance between a bathymetric profile and a section of the DEM. When this criteria is a sum of absolute differences, (3) becomes:

$$E(|T - Z|) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} |T_{ij} - Z_k| \cdot p_{ijk} \tag{4}$$

The remaining problem addresses computation of the joint probability:

$$p_{ijk} = Pr(x_i = x_k, y_j = y_k)$$

This probability represents the compatibility of the two distributions (x_i, y_j) on one hand and (x_k, y_k) on the other hand.

Two approaches are proposed to compute this probability:

- a purely statistical approach using Mahalanobis distance,
- a fuzzy approach based on generalization of error calculus.

B. STATISTICAL APPROACH

Statistical approach considers that each cell (i,j) of the (x,y) plane as a uniform probability density with mean (x_i, y_j) and spread (dx, dy) (Figure 2).

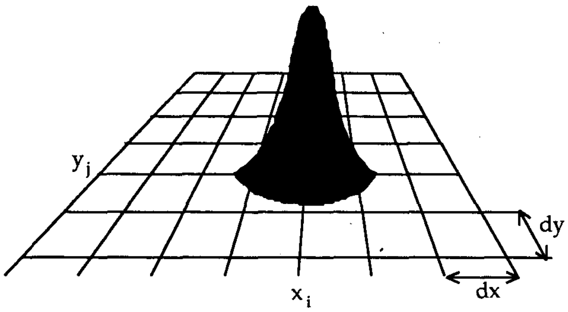

Figure 2. Probability density

The variance/covariance matrix of the vector $[x_i,y_j]^T$ is:

$$\Lambda_{ij} = \begin{bmatrix} \sigma_{x_i}^2 & 0 \\ 0 & \sigma_{y_j}^2 \end{bmatrix} \tag{5}$$

The position vector $[x_k,y_k]^T$ of the measured point P_k is a function of the sensor location parameters (X_0, Y_0, P, Ψ). If the errors on this expected location, and on the measurement (ρ_k, θ_k) are small enough and not correlated, the variance/covariance matrix of this position vector $[x_k,y_k]^T$ can be approximated by:

$$\Lambda_k \approx \begin{bmatrix} \frac{\partial x_k}{\partial X_0} & \cdots & \frac{\partial x_k}{\partial \theta} \\ \frac{\partial y_k}{\partial X_0} & \cdots & \frac{\partial y_k}{\partial \theta} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{X_0}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{Y_0}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\Psi^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\rho^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\theta^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial x_k}{\partial X_0} & \frac{\partial y_k}{\partial X_0} \\ \vdots & \vdots \\ \frac{\partial x_k}{\partial \theta} & \frac{\partial y_k}{\partial \theta} \end{bmatrix} \tag{6}$$

$$\Lambda_k \approx \begin{bmatrix} \sigma_{x_k}^2 & \sigma_{xy_k}^2 \\ \sigma_{xy_k}^2 & \sigma_{y_k}^2 \end{bmatrix} \tag{7}$$

$$\begin{cases} \sigma_{x_k}^2 = \sigma_{X_0}^2 + \rho_k^2 \cos^2(\Psi)\sin^2(\theta_k)\sigma_\Psi^2 + \sin^2(\Psi)\sin^2(\theta_k)\sigma_\rho^2 + \rho_k^2 \sin^2(\Psi)\cos^2(\theta_k)\sigma_\theta^2 \\ \sigma_{y_k}^2 = \sigma_{Y_0}^2 + \rho_k^2 \sin^2(\Psi)\sin^2(\theta_k)\sigma_\Psi^2 + \cos^2(\Psi)\sin^2(\theta_k)\sigma_\rho^2 + \rho_k^2 \cos^2(\Psi)\cos^2(\theta_k)\sigma_\theta^2 \\ \sigma_{xy_k}^2 = \cos(\Psi)\sin(\Psi)\sin^2(\theta_k)\left(\rho_k^2\sigma_\Psi^2 + \sigma_\rho^2\right) - \rho_k^2 \cos(\Psi)\sin(\Psi)\cos^2(\theta_k)\sigma_\theta^2 \end{cases}$$

where $\sigma X0$, $\sigma Y0$, σ_ψ are the variances associated with discretization of the search space. σ_ρ is the variance of the measurement error. σ_θ is the variance associated with the beam positioning error and the discretization of θ.

Mahalanobis distance between $[x_i,y_j]^T$ and $[x_k,y_k]^T$ is thus given by:

$$M = \begin{pmatrix} x_i - x_k \\ y_j - y_k \end{pmatrix}^T \cdot \left(\Lambda_{ij} + \Lambda_k\right) \cdot \begin{pmatrix} x_i - x_k \\ y_j - y_k \end{pmatrix} \tag{8}$$

Then joint probability p_{ijk} can be estimated with the Mahalanobis coefficient :

$$Pr\left(x_i = x_k, y_j = y_k\right) = e^{-M} \tag{9}$$

C. Fuzzy Approach

In fuzzy approach, error is considered as a problem of precision. Any variable can be considered as a fuzzy interval whose shape is most commonly trapezoidal. It is also the most *neutral* one.

A "fuzzy telling" represents quantization of the DEM. Distributions x_i and y_j are supposed to be non-interactive. Thus each cell is associated with a pyramid-shaped fuzzy box (Figure 3). A fuzzy box represents the Cartesian product of two fuzzy intervals. The support (respectively the core) of the pyramid is a rectangle centered at (x_i,y_j) with spread $(2.dx,2.dy)$ (respectively (dx,dy)).

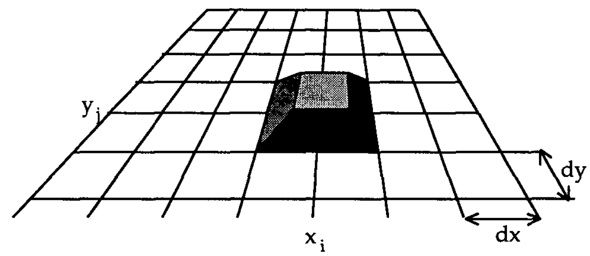

Figure 3. Fuzzy subset associated with (x_i,y_j).

The fuzzy subset associated with the position (x_k,y_k) of the k[th] impact of the sonar beam is also approximated by a pyramid. Advanced error calculus [9] is used to provide both support and core. Transitions of belong function between support and core are supposed to be linear.

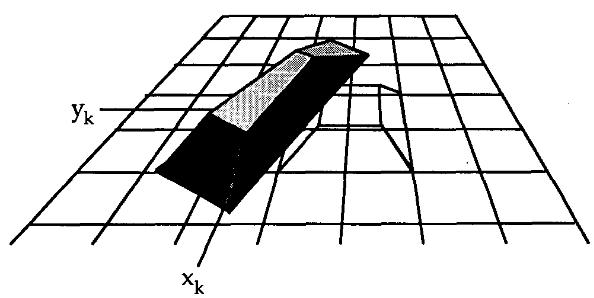

Figure 4. Fuzzy subsets intersection.

Then, possibility of interaction between each cell (i,j) with the measure (k) is evaluated by intersecting the fuzzy subset associated with the measure and the fuzzy box associated with the cell (i,j) (Figure 4). Therefore, a possibility distribution π_{ijk} is obtained :

$$\pi_{ijk} = \Pi(x_i = x_k, y_j = y_k) \tag{10}$$

This possibility distribution is then transformed into a probability distribution using the classical method described in [10].

IV. TRACKING

A tracking process based on Kalman filtering is used to reduce the search area during each new localization step. In the general case, this search area is a 4 dimensional space (X_0, Y_0, ψ, P). Figure 5 represents this area in case of a 3D space, when the depth parameter P is assumed to be constant. The search area is discretized according to the precision required by the localization task. For each cell of the search area we compute a similarity criteria between the DEM and the sonar measurements, according to equation (4). The most likely localization is supposed to be the one that minimizes this criterium.

Variances σ_{X0}, σ_{Y0}, σ_p and σ_Ψ required in equation (7) are computed as functions of the parameter space discretization [11]. For example:

$$\sigma_X = \frac{dx}{\sqrt{12}} \approx \frac{dx}{3{,}46} \qquad (11)$$

where dx is the quantization unit (figure 2).

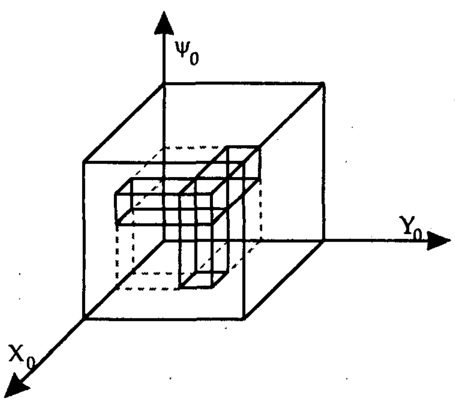

Figure 5. Search area in a 3D space.

So, the accuracy of the localization depends on the discretization of the search space. If the size of the unit cell is reduced, the precision increases together with the computation time. Consequently, the number of cells of the search area must be limited. However, if the search area is too small, it would not include the expected value of the localization.

To reduce the influence of the parameter space discretization, a Kalman filter [12] is used to predict the vehicle localization. It provides a smooth trajectory and it allows us to have a better estimate of the center of the search area at each step.

V. EXPERIMENTAL RESULTS

A real reference map which is a digital elevation model (DEM) of the Var underwater canyon covering a 27km by 27km zone (Figure 6) is used for this experimentation. The sampling interval for X and Y is 100m. We define a complex trajectory on this map (Figure 7) in a plane located at a depth P=500m. We simulate the data acquisition along this trajectory with the sonar model defined in section II. The depth being constant, the localization space used for matching is a 3 dimensional space defined by parameters X, Y and ψ.

The search space is arbitrarily quantized in cells whose size is 100m for X and Y and 0.2rad for ψ. The search area includes N_x*N_y*N_ψ cells. We arbitrarily choose $N_x=N_y=N_\psi$.

We present the trajectory estimations obtained with the two matching algorithms described in sections III-B and III-C. The tracking is achieved with an extended Kalman filter, which uses non linear equations to model the vehicle motion.

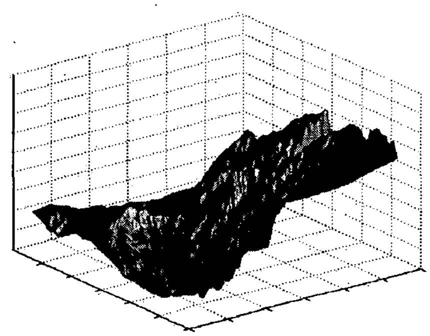

Figure 6. DEM of the Var underwater canyon

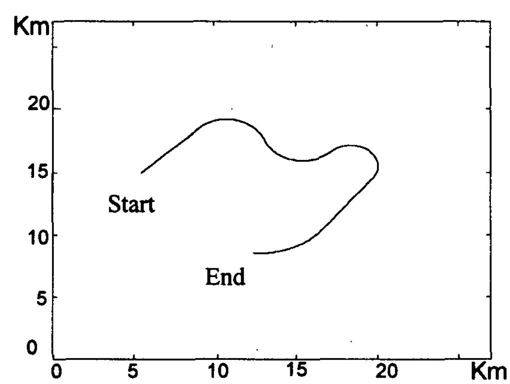

Figure 7. Sonar trajectory

A. Mahalanobis distance

Table 1 presents the average time required to process a bathymetric profile by using the statistical approach. These results have been obtained with a Pentium II (333 MHz). They show that the computation time increases significantly with the number of cells of the search area.

Table 1. Average processing time with the statistical algorithm.

$N_x*N_y*N_\psi$	4*4*4	5*5*5	6*6*6
Processing time	0.2 sec	0.39 sec	0,78 sec

Moreover, we note that when this number is smaller than 6*6*6, the tracking process fails (Figures 8,9). The reliability of the tracking assigns lower limits to the search area dimensions.

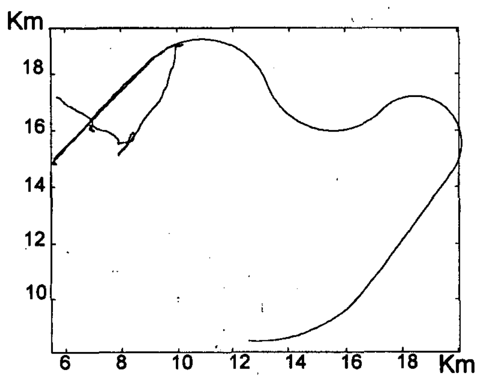

*Figure 8. Tracking in a 4*4*4 search area, with the statistical algorithm.*

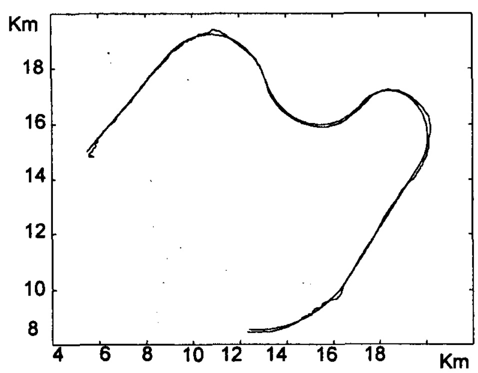

*Figure 9. Tracking in a 6*6*6 search area, with the statistical algorithm.*

Table 2 gives the maximum error E_{max} and the average error E between the estimated and the simulated trajectory presented in figure 9 (when the dimension of the search space is 6*6*6). The average error is always smaller than the sampling interval in each direction about 80m (with regard to 100m) for X and Y coordinates and 0,13rad (with regard to 0,2rad) for ψ. These results are within the specifications of a subsea navigation task.

Table 2. Accuracy of the estimated trajectory, with the statistical algorithm

Parameter	X	Y	ψ
E_{max}	481m	274m	0,48 rad
E	83m	78m	0,13 rad

This matching algorithm based on a statistical approach provides accurate localization (average errors are smaller than the sampling interval) with fast computation (time is less than 1s to process a bathymetric profile). However this method suffers from a lack of robustness with respect to some input parameters such as the number and the size of the quantization intervals of the search space. This is due to the fact that statistic methods provide bad results with small data sets.

B. Fuzzy approach

We now present the experimental results obtained with the fuzzy approach. In order to compare the results of both methods, we use the same trajectory (figure 7).

Table 3. Average processing time with the fuzzy algorithm

$N_x*N_y*N_\psi$	4*4*4	5*5*5
Processing time	7,9 sec	14,7 sec

Table 3 presents the average time required to process a bathymetric profile by using the possibility theory. Once again this time doubles when one interval is added to each axis of the search area. We note that this time is greater than with the previous matching algorithm. About 8 seconds are necessary to find the best match in a (4*4*4) search area (instead of 0,2 sec).

Table 4 gives the maximum error E_{max} and the average error E between the estimated and the simulated trajectory, for a 4*4*4 and a 5*5*5 search space. Once again, the average errors are smaller than the sampling interval (about 65m (with regard to 100m) for X and Y coordinates and 0,11rad (with regard to 0,2rad) for ψ). We also note that whatever the dimension of the search area may be, the tracking algorithm never fails. For instance, on figure 10, the search area includes only 4*4*4 cells.

Table 4. Accuracy of the estimated trajectories with the fuzzy algorithm

$N_x*N_y*N_\psi$	Parameter	X	Y	ψ
4*4*4	E_{max}	221m	249m	0,33rad
	E	66m	64m	0,12rad
5*5*5	E_{max}	273m	209m	0,32rad
	E	64m	64m	0,44rad

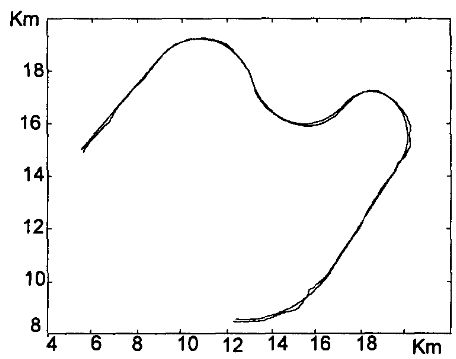

*Figure 10. Tracking in a 4*4*4 search area, with the fuzzy algorithm.*

This matching method based on the possibility theory is more accurate than the one using Mahalanobis distance. The average error is 65m for X, Y instead of 80m and 0.11rad for ψ instead of 0.13rad. Moreover, this method is more robust to changes in search area dimension. Only a 4*4*4 area is needed to perform a good tracking of the reference trajectory. However, the drawback of this method is the processing time. It can be a problem for real-time applications.

VI. Conclusion

In this paper, we have presented a terrain-referenced localization method based on the matching of rough sonar data with a digital elevation map. The correlation function takes into account the non-regular sampling of the sonar data in the reference space. Two matching algorithms, based respectively on a statistical and on a fuzzy approach, have been implemented and compared. Moreover, a significant decrease of the computation time is obtained by using a Kalman filter to predict the motion of the sensor, in order to reduce the dimension of the search zone.

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References

[1] J. J. Rodriguez, J. K. Aggarwal. *Matching Aerial Images to 3D Terrain maps.* IEEE Transaction on P.A.M.I., vol. 12, n° 12, December 1990, pp. 1138-1149.

[2] C. F. Olson, L.H. Matthies, *Maximum Likelihood Rover Localization by Matching Range Maps*, IEEE Int. Conference ICRA'98, Leuven, Belgium, May 1998 pp. 272-277.

[3] J. Banks, M. Bennamoun, P. Corke. *Fast and robust stereo matching algorithms for mining automation.* JAF 1997, Adelaïde, Australia, pp. 139-149.

[4] S. Wasielewski and M. J. Aldon, *Dynamic vision for ROV stabilization*, IEEE Int. Conference Oceans'96, Fort-Lauderdale, USA, September 1996, pp. 1082-1087.

[5] S. Daniel, F. Le Léannec, C. Roux, B. Solaiman, E. P. Maillard, *Side-Scan Sonar Image Matching*, Ocean Engineering, Vol. 23, n° 3, pp. 245-259.

[6] P. Newman, H. Durrant-Whyte, *Using Sonar in Terrain_Aided Navigation*, IEEE Int. Conference ICRA'98, Leuven, Belgium, May 1998, pp. 440- 445.

[7] L. Lucido, J. Opderbecke, V. Rigaud, R. Deriche, Z. Zhang, *A Terrain Referenced Underwater positioning Using Sonar Bathymetric Profiles and Multiscale Analysis*, IEEE Int. Conference Oceans'96, Fort-Lauderdale, USA, September 1996.

[8] M. Sistiaga, J. Opderbecke, M. J. Aldon, V. Rigaud, *Map-Based Underwater Navigation Using a Multi-beam Echosounder*, IEEE Int. Conference Oceans'98, Nice, France, 28 Sept. – 1 Oct. 1998

[9] E. Walter and L. Pronzato, *Identification of parametric models from experimental data*, Communications and Control Engineering Series, Springer, London, 1997.

[10] D. Dubois, H. Prade, *Possibility theory*, Plenum Press, London, 1988.

[11] B. Kamgar-Parsi and B. Kamgar-Parsi, *Evaluation of Quatization Error in Computer Vision*, IEEE Transastion on P.A.M.I., vol. II, n° 9, September 1989, pp. 929-939.

[12] A. H. Jazwinski *Stochastic Process and Filtering Theory,* Academic Press, New-York, 1970.