REDUCING THE PRECISION / UNCERTAINTY DUALITY IN THE HOUGH TRANSFORM

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ABSTRACT
The Hough transform is a popular method for detecting complex forms in digital images. However, the technique is not very robust since several parameters that determine the scope of the detection results, such as quantization thresholds and intervals, must first be defined. In the present paper, we propose to enhance shape detection with the Hough transform through fuzzy analysis. One chief drawback of the Hough transform, i.e. the uncertainty / precision duality, is thus reduced.

I. INTRODUCTION
Since the paper by Hough [HOU 62] presenting a transform that aimed to detect sets of linear points in noisy binary images, several improvement of the Hough transform (HT) have been proposed.

The method and its extensions have been reviewed in detail [MAI 85], [ILL 88] and an exhaustive bibliography is available [PIC 87]. In particular, many papers have focused on the effect of errors due to random noise and data quantization [HUN 90] [STE 91] [PRI 94].

Despite the large number of papers dedicated to the Hough Transform, relatively little attention has been paid to what we termed the "uncertainty/precision duality." This duality could be set out as follows: as shape detection precision increases, the reliability of the detection decreases. This seems to be due to the binary aspect of the vote in the classical Hough transform (CHT).

Han and al. [HAN 93] proposed to use fuzzy subset theory to deal with the problem of approximate concepts in HT. They designed a Fuzzy Hough Transform (FHT) which generalizes the distributed voting principle described by Thrift and Dunn [THR 83]. However, this method makes no distinction between data uncertainty and expected or computationally–induced parameter uncertainty. In addition, there is no benefit from assessing the data in terms of confidence.

In this paper we present a new FHT approach that takes current knowledge on uncertainty into consideration to improve shape detection.

II. THRESHOLDING
II.1. EFFECT OF THRESHOLD
Because it was designed for binary images, using HT is not problem-free on grey-level images. Users first have to face the problem of defining the subset of image points that supposedly belong to the sought–after straight lines (generally by thresholding the image). An image that is either non-uniformly illuminated or ill-contrasted requires to over- or under-estimate the threshold. Which induces an uncontrolled bias on the HT. Statistically finding an “optimal” threshold [JOL 89] supposes to test each pixel for two hypotheses that are: “the pixel belongs to one of the sought-after lines” and “the pixel belongs to none of the sought-after lines”. Those two hypotheses are used to define two thresholds. Those two thresholds are used in the FHT to define the fuzzy subset of the points in the image that are supposed to belong to one of the searched lines.

II.2. FUZZY THRESHOLDING
Image pixels can generally be separated into two subsets E and E according to grey level, gradient and curvature properties. E is the subset of points belonging to one of the sought–after straight lines, and its complement is the subset of points that belong to no straight line.

Many solutions have been proposed to overcome this problem, including that of [O’GO 73], whereby grey–level values are used directly to weight HT votes.

We use a similar technique to define E as a fuzzy subset of the original image.

When searching for black lines on a white background, the membership \( \mu_q(x_i,y_j) \) of a pixel \((i,j)\) with a grey level \( g(i,j) \) to a fuzzy subset \( E \) could be defined from an \( \mu \) function by:

\[
\text{Erreur}!
\]

When \( I_{pq} \), the crisp subset of \( I \), is denoted by:

\[
I_{pq} = \{(x,y) \in I | x \cos(\theta_q) + y \sin(\theta_q) - p_q \leq \delta_q \}
\]
then, in assessing the $P_p \times \Theta_q$ cell, the HT is equal to the fuzzy cardinal of the subset $E \cap I_{pq} = I_{pq}$.

$$h(P_p \times \Theta_q) = \text{CARD}(E_{pq}) = \text{Erreur} !$$

In practice, when the image grey–level distribution is unknown, it is better to use a linear function:

$$L(u) = 1 - u \text{ if } u \in [0, 1]$$

### III. QUANTIZATION
#### III.1. EFFECT OF QUANTIZATION

In practice, using HT implies subdividing the parameter space into a certain number of cells. Many papers have rightly focused on this problem of quantization. There are two objectives in adjusting quantization:

- enhanced computation performance (less storage memory required, reduced computation time),
- enhanced algorithm performance (precision, confidence).

The first objective reduces the cell number (increased quantiation), while the status of the second is not as clearcut, and is the focus of the present study.

In order to increase line detection precision, quantization clearly has to be reduced. This reduction can also be motivated by a high density of lines in the image. In addition to a substantial increase in computation time, there will be a lower accumulator coefficient, and therefore a larger uncertainty in the detection of each straight line. When this phenomenon is exaggerated with an excessive quantization, the accumulator array will only contain a maximum of one point per cell.

Conversely, if we intend to tolerate a poor fit between the model and reality, then it would be better to increase quantization. This increases the values of the accumulators associated with each cell, thus increasing the confidence that straight lines are present in the image. However, enhancing the certainty in the detection of each line will increase the imprecision of this detection.

The main reason for this dichotomy between the expected line detection precision and the certainty that the line is present on the image is the binary aspect of the vote in CHT.

Fuzzy voting scheme is a good way of approaching the quantization difficulties in Hough transform. It consists of distributing the votes in the parameter space in an uncertainty zone defined by the fuzzy approach. This avoids the all-or-none aspect of the usual integration of the accumulator associated with each cell in parameter space. Then, the quantization no more affects precision but only the resolving power of the transform i.e., its ability to create two different peaks for two different close features.

### III.2. DISTRIBUTED VOTE

In a classical Hough transform (CHT) [CHA 91], each pixel $(i,j)$ is considered as an intersection point of coordinates $(x_i, y_j)$ on the image $I \subset \mathbb{R}^2$. The parameter space is subdivided into $(2.\delta p, 2.\delta \theta)$–sized boxes $P_p \times \Theta_q$ centered on $(\rho_p, \theta_q)$.

Each pixel $(x_i, y_j)$ gives rise to a sine curvature in $\Omega$ space. The Hough transform is assessed by incrementing, for each characteristic point (i.e. $(x_i, y_j) \in E$), accumulators associated with $\Omega$ boxes that intersect the sine curvature in a non–null manner (FIG. 1).

![FIG 1: Sine curvature fitted to an image point $(x_i, y_j)$](image)

In practice, this means calculating the $\rho$ values matching each $\theta_q$ value of $\theta$: $\rho_{pq}(x_i, \cos \theta_q + y_j, \sin \theta_q)$ and incrementing the $P_p \times \Theta_q$ cell accumulator, such that: $\rho \in [\rho_p - \Delta \rho, \rho_p + \Delta \rho]$.

To account for uncertainty in detecting characteristic points, it is necessary to consider each pixel $(i,j)$ as a $(2.\delta x, 2.\delta y)$–sized box $X_{i}X_{j}$ of the image $I \subset \mathbb{R}^2$ centered on $(x_i, y_j)$.

A box $X_iX_j$ of $I$ maps a sine curvature set in parameter space. It is thus essential to increment accumulators associated with all boxes that intersect at least one sine curvature of this set in a non–null manner. This is termed the distributed vote.

![FIG 2: Sine curves fitted to an image box $X_iX_j$](image)

A first–order approximation can simplify this intersection calculation. Hence, for each $\theta_q$ value, we evaluate the interval $[\rho - \Delta \rho, \rho + \Delta \rho]$ , such that:

$$\rho = (x_i \cos \theta_q + y_j \sin \theta_q)$$

$$\Delta \rho = 2.\delta x.\cos \theta_q + 2.\delta y.\sin \theta_q + (y_j \cos \theta_q - x_i \sin \theta_q)\delta \theta$$

The $P_p \times \Theta_q$ cell accumulator is then incremented, for which:
\[ \rho \in [\rho_0 - \Delta \rho, \rho_0 + \Delta \rho] \cap [\rho_0 - \rho p, \rho_0 + \rho p] \neq \emptyset. \quad (1) \]

Equation (1) states that \((x,y,\rho,\theta) \in X_i Y_j X P \times \Theta q\) exists, such that \(\rho = (x \cos \theta + y \sin \theta)\), indicating that the relation \(f(x,y,\rho,\theta) = 0\) is entirely possible in this box.

The all–or–none aspect of this incrementation can be almost reduced by incrementing the accumulators associated with the \(P_p \times \Theta q\) cell using a proportional value, whereby the \([\rho_0 - \Delta \rho, \rho_0 + \Delta \rho]\) interval overlaps each \(P_p\) interval. The fuzzy Hough transform normalizes this heuristic process.

**III.3. FUZZY VOTE**

Let us consider that each pixel (i,j) is a fuzzy box, i.e. a product of two fuzzy intervals \(X_i\) and \(Y_j\). \(X_i\) (resp.\(Y_j\)) is a symmetrical fuzzy interval centered on \(x_i\) (resp.\(y_j\)) with kernel \([x_i - \delta x, x_i + \delta x]\) (resp.\([y_j - \delta y, y_j + \delta y]\)) and spread \(\sigma_x\) (resp.\(\sigma_y\)) (FIG. 3).

\[ \text{FIG 3 : Fuzzy pixel (fuzzy image box)} \]

Therefore, the sine curve set produced by each pixel in \(\Omega\) is an induced fuzzy set (FIG. 4).

\[ \text{FIG 4 : Fuzzy sine curve set fitted to a fuzzy image box } X_i Y_j. \]

Each \(\Omega\) cell is stated in the same manner as the Cartesian (fuzzy) product of two fuzzy intervals \(P_p\) and \(\Theta q\). \(P_p\) (resp.\(\Theta q\)) is the fuzzy interval with a kernel \([p_0 - \rho p, p_0 + \rho p]\) (resp. \([\theta_0 - \theta_0, \theta_0 + \theta_0]\)), and spread \(\sigma_p\) (resp.\(\sigma_q\)).

We find that, for each \(X_i Y_j\) box of \(E\), the relation \(f(x,y,\rho,\theta) = 0\) is possible for \(P_p \times \Theta q\) cells with incremented accumulators. This potential is shown by the fact that the value 0 belongs to the variation domain of \(\phi = f(x,y,\rho,\theta)\) when variables \(x,y,\rho\) and \(\theta\) are restricted by their variation domain.

Variation domains in the FHT are fuzzy. Hence, when \((x,y,\rho,\theta)\) belongs to \(X_i Y_j X P \times \Theta q\), the range of the variable \(\phi\) is the fuzzy variation domain \(\Phi_{ijpq}\). According to the extension principle, when \(\mu_{\Phi_{ijpq}}\) is the membership function of \(\Phi_{ijpq}\), we can state:

\[
(\phi) = \text{SUP}_{x \in X, y \in Y, \rho \in P, \theta \in \Theta} \left\{ \text{MIN} \left( \mu_x(x), \mu_y(y), \mu_p(p), \mu_{\Theta q}(\theta) \right)/ f(x,y,\rho,\theta) \right\}
\]

The possibility of membership of a pixel \((i,j)\) to the fuzzy line represented by the cell \((p,q)\) is clearly membership of the value 0 to the fuzzy set \(\Phi_{ijpq}\):

\[ \pi(i,j,p,q) = \mu_{\Phi_{ijpq}}(0) \]

FHT assessment of the fuzzy cell \(P_p \times \Theta q\) is then defined, for all fuzzy pixels of \(E\), as the sum of this occurrence possibility \(\pi(i,j,p,q)\). Replacing the fuzzy boxes by crisp boxes will clearly lead to a classical HT.

Now, to evaluate \(\mu_{\Phi_{ijpq}}(\phi)\), we will use the properties of fuzzy intervals in an LR representation. The fuzzy intervals used here are for obvious reasons considered to be symmetrical. According to the properties of fuzzy intervals, the fuzzy interval \(\Phi_{ijpq}\) can be defined by:

\[ \Phi_{ijpq} = P_p - X_i \cos(\Theta q) - Y_j \sin(\Theta q). \]

In consideration of a few restrictions specific to limited developments, \(\Phi_{ijpq}\) can be mapped with an LR–type interval, as defined by its center \(\phi_{ijpq}\) the half–width of its kernel \(\delta \phi_{ijpq}\) and its spread \(\sigma\).

\[ \text{ERROR !} := \sigma \sigma_p + |\cos \Theta q| \sigma_x + |\sin \Theta q| \sigma_y + \text{ERROR !} \cdot \sigma_0 \]

The FHT assesses the cell \(\{p,q\}\) as follows:

\[ \hat{h}(P_p \times \Theta q) = \text{ERROR !} \cdot \mu \text{ERROR !} = \text{ERROR !} \cdot \pi(i,j,p,q) \]

The latter equation still has to be corrected to account for the fact that \(E\) is a fuzzy set of \(I\) with a membership function \(\mu_{\Phi}(X_i Y_j)\):

\[ \hat{h}(P_p \times \Theta q) = \text{ERROR !} \]

**IV. DEFFUZZIFICATION**

Defuzzification of the HT involves retrieving the distributed vote for a line of a cell from the neighboring cells by calculating the barycenter of the concerned fuzzy subsets \([F_{OU} \ 92]\). This last procedure improves the accuracy. This accuracy is no more linked to the quantization in parameter space. Certainty is related to the relative height of the peaks.

**V. EXPERIMENTS**

We run the experiments on a simple image in order to highlight the different specificity of the FHT versus the CHT. This image has been chosen because it presents most of the flaw the FHT is coping with:
grainy and non-uniform background, poor contrasts, fuzzy unclear lines, ....

![Image](image_url)

**FIG. 5**: Original image of three cables.

The combined effect of fuzzy thresholding and fuzzy voting reduce the problems of false peaks due to noise in the image (FIG. 6). There are very few noise peaks on the accumulator array for the FHT, in contrast to that of the CHT. This could be explained by joint effects of a distributed vote and fuzzy weighting, thus “smoothing” the transform.

![Image](image_url)

**FIG. 6**: Accumulator arrays for the CHT and the FHT.

![Image](image_url)

**FIG. 7**: Detected straight lines superimposed on the original image.

![Image](image_url)

**FIG. 8**: Poor quantization of the parameter space.

![Image](image_url)

**FIG. 9**: Poor gray level threshold.

**V. CONCLUSION**

Overall, the results of several tests showed that when the analyzed image is crisp, well contrasted, with little noise and quantization is adequate, then there is no visible improvement in the quality of straight line detection in the image when using the FHT. However, in the absence of any of these characteristics, there is a clear detection degradation with the CHT but not with the FHT. The FHT is therefore more robust than the CHT.


