FILTERING AND FUSING COMPASS AND GYROMETER DATA USING GUESS FILTER

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ABSTRACT

Most papers dealing with data fusion try to use a single tool to provide the "best" estimation. But, as pointed out by Prade and Dubois [DUB 88], most tools dealing with imperfect information aim at different purposes. In a previous paper [STR 96], the Guess filter has been presented. This filter aims at using three different error theories together to obtain an estimation that combines robustness, accuracy, reliability and easy setup. Possibility theory handles precision, statistical theory is used to reduce uncertainty, and rough sets theory allows a robust and easy computation of the resulting filter. In the Guess filter, data are represented as a possibility distribution. Because of this representation, data issued from different sensors can be combined at both high and low level. Fusion at a low-level takes advantage of redundancy to reduce the overall uncertainty and thus to increase accuracy. Fusion at a high-level reduces the influence of inadequacy in data modeling. This method has been implemented on a submarine robot. Experimental results are presented.

I. INTRODUCTION

Even if a great amount of papers has been written on multi-sensor fusion, the problem still seems to be open. Actually, the proliferation of inexpensive sensors and the increasing complexity of the tasks to be executed by the robots lead to the need for an accurate and reliable information provided by perception systems.

The so-called data fusion process is a suitable way of improving both accuracy and reliability of sensor information. It consists of combining several measurements provided by different sensors (or by the same sensor at different instants) into a unique assessment. This process leads to the reduction of the intrinsic limitation of each sensor. Therefore, a key-point of every data fusion process is error handling.

What is error? According to signal processing, it is *something that is not the information to be recovered.* In fact, error in measurement can be divided into three basic categories: uncertainty, inaccuracy and inconsistency.

If a measurement is corrupted by uncertainty, it means that there is a lack of certainty for this measure to be in accordance with the information to be recovered. As an example, if the measurements provided by an accelerometer vary although the sensor is not moving, then these measurements are uncertain.

On the contrary, an inaccurate measurement is in perfect accordance with the information under consideration. The lack of precision is due to sensor limitations. For instance, the precision of a measurement provided by a LASER range-finder is limited by the fact that electronic processes are used to measure light speed information.

Finally, inconsistency refers to the usefulness of a sensor to get an information due to its environment or to its own limitation. For example, an inclinometer cannot give an information related to its inclination when this inclination exceeds its measurement range. It is also unable to give a proper measurement when the robot undergoes an acceleration.

Data fusion seems to be able to cope with most of those problems. In fact, a consensus between several sensors can reduce uncertainty. Precision can be increased by combining the domain of several measurements. Moreover, limitation of a sensor can be palliated by the aptitude of another sensor.

Several mathematical tools are available to handle errors in measurements. During the last ten years, probabilistic theory has been widely used to perform data fusion. The core of those techniques is to use the past knowledge about the occurrence of an event to infer the chance of occurrence of a similar event in the future. As voting procedures, they are rather concerned by uncertainty. Conversely, set-based techniques deal with (un)precision. In this framework, measurements are considered as sets containing the information. Handling measurements consists of set operations like union, intersection or volume approximation. Union provides the most reliable set while intersection provides the most accurate set that is supposed to include all or part of the information to be recovered.

Demster-Shaffer belief theory provides a formalism able to deal with both sets and confidence measurement. As a drawback, using this technique does not allow to handle properly situations where severe conflict occur. Moreover, knowledge on data has to be available in numerical values.

Fuzzy sets seem to be a more general framework to deal with precision and certainty. Indeed, a fuzzy sub-set can represent a precision domain while possibility

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theory, that ensue from fuzzy sub-set theory, deals with uncertainty. Moreover, fuzzy logic links semantic space to numerical space.

Fuzzy sub-sets have rather been used for high-level process [ABI 91] while statistics continue to dominate much of the work on low-level sensing processes. The main reason is that, in a statistical framework, data are fairly represented by a mean vector and a covariance matrix, that are easily manipulated through matrix computation. The problem that arises is the difficulty to infer high- and low-level processes.

The present paper proposes a solution to combine both high- and low-level processes by using a filter based on possibility theory. This new filtering process has been experimented to estimate the heading of an underwater robot by fusing compass and gyrometer data.

II. FILTERING WITH POSSIBILITY

Let $\omega(t)$ be a time varying signal to be estimated. Let S be a sensor that provides an uncertain, inaccurate and discrete measurement $m_k=m(kT)$ ($k\in\mathbb{N}$). Because of both inaccuracy and sampling, m_k has to be considered as a subset M_k (generally an interval) of the set Ω of the possible values of ω.

Filtering with possibility consists of finding a possibility distribution $\pi_k(\omega)$ on Ω, i.e., a fuzzy subset Θ_k of the possible values of ω at time k.

First, let us suppose that $\omega(t)$ is stationary: $\omega(t)=C$. Estimating C consists of finding a subset Θ_k such that $C\in\Theta_k$. From a set-theoretic point of view, there are two extreme modes of combination depending on the reliability of the sensor.

If the sensor is fully reliable, then $\forall k$, $C\in M_k$. Therefore, $C\in\Theta_k=\bigcap_{i=1\ldots k} M_i$. Θ_k is the most accurate estimation of C (the smallest subset). Conversely, if the sensor is not reliable, $\Theta_k=\bigcup_{i=1\ldots k} M_i$. is the most reliable estimation of C. The confidence increases with k. The disjunctive mode of combination increases reliability while precision decreases. The dual effect is obtained by conjunctive mode.

Now, if the a priori given model for the evolution of $\omega(t)$ is not reliable or varies with time, pure set operation can no longer be used and must be replaced by a statistical estimation. The influence of the information given by the measurement M_{k-n} for estimating Θ_k has to weaken when n increases.

So, filtering the measurements consists of a dissymmetric combination process. The new measurement M_{k+1} has to be used to modify the a-priori knowledge on Θ_k i.e. to revise $\mu_{\Theta_k}(\omega) = \pi_k(\omega)$ in the light of $\mu_{M_{k+1}}(\omega)$.

In [DUB 92], several symmetric and dissymmetric aggregation rules are proposed. The core of those techniques is to find a way of changing gradually the combination mode from conjunction to disjunction according to a measure of the conflict between the measurements. According to possibility theory, this conflict can be measured by the conditional possibility and conditional necessity:

$$\Pi(\Theta_k | M_{k+1}) = \sup_{\theta\in\Omega}\left\{\min\left(\mu_{\Theta_k}(\theta), \mu_{M+1}(\theta)\right)\right\} \tag{1}$$

$$N(\Theta_k | M_{k+1}) = \inf_{\theta\in\Omega}\left\{\max\left(\mu_{\Theta_k}(\theta), 1-\mu_{M+1}(\theta)\right)\right\} \tag{2}$$

In [DEV 93], an application of the more elaborate symmetric aggregation rule has been implemented for filtering. Its performance has been qualitatively compared with that of a Kalman filter. If the signal is fairly corrupted by contaminated noise, the conflicts seem to be ill-handled and this filter is rather unstable. This is due to the use of a symmetrical rule to perform a dissymmetric problem: the same weight cannot be given to the a priori knowledge and the new measurement.

To overcome these problems, we propose to combine a voting process with a non voting process and to avoid normalization of the possibility distribution. The voting part of the process is performed by a first order statistical process. This new process allows the representation of confidence and uncertainty knowledge, at two different levels. It is performed in two steps.

First, a symmetrical combination rule is applied to provide a deduced set D_{k+1}:

$$\begin{aligned}\mu_{D_{k+1}}(\theta) &= \min\left(\min\left(\mu_{\Theta_k}(\theta), \mu_{M_{k+1}}(\theta)\right), \eta\right) \\ &+ \min\left(\max\left(\mu_{\Theta_k}(\theta), \mu_{M_{k+1}}(\theta)\right), 1-\mu_{\Theta_k}(\theta), 1-\eta\right)\end{aligned} \tag{4}$$

where

$$\begin{aligned}\eta &= \Pi(\Theta_k | M_{k+1}) + N(\Theta_k | M_{k+1}) \\ &- \Pi(\Theta_k | M_{k+1}).N(\Theta_k | M_{k+1})\end{aligned}$$

is an estimate of the conflict between the prediction and the measurement.

Then, D_{k+1} is used to update Θ_k to give the final distribution:

$$\mu_{\Theta_{k+1}}(\theta) = \alpha.\mu_{\Theta_k}(\theta) + (1-\alpha).\mu_{D_{k+1}}(\theta) \tag{5}$$

with $\alpha\in[0,1]$.

This last update is performed by a first order exponential statistical filter on the possibility distribution instead of being applied on the value itself. α can be fixed a priori if the noise has been clearly identified or be updated by a fuzzy rule or Baye's rules depending on the a priori knowledge available on the process.

A problem still remains: computation of such rules leads to the need for an appropriate data structure.

III. COMPUTING WITH ROUGH SETS

Two classical ways are mainly available to perform the computation of this filter. First of all, the fuzzy set can be considered as a union of fuzzy intervals. The second possibility consist of sampling the membership function on Ω. Let us see the advantages and the drawbacks of each method.

Assuming that the measurements are fuzzy intervals, the union-like and intersection-like operations performed in the filtering process also provide fuzzy intervals. The advantage of such a representation is accuracy. The drawback is that the computation time highly depends on the number of elements of the list. This limitation is not acceptable if the filter is to be implemented on a real process. Moreover, non-consistent intervals have to be removed in order to avoid an exponential memory request. If such rules are used, then the benefit of accuracy of this representation vanishes. Otherwise, the memory can explode.

Hence, a sampled representation is generally preferred. It consists of assuming that the fuzzy membership can be represented by membership function on sparse elements of Ω. In fact, this representation is an incomplete and imprecise representation of this membership function. Moreover, the data structure does not take into account this coarsening.

In order to deal with this rough representation and handle computation in a proper manner, we propose to use the recent theory of fuzzy rough sets proposed by Dubois and Prade in [DUB 90]. This will be performed by decomposing the possibility distribution on a fuzzy partition of Ω.

First of all, let us have a brief overview of fuzzy rough sets. Fuzzy sets and rough sets both deal with imprecision. But while the poor definition of boundaries of sub-classes are properly modeled by fuzzy sets, rough sets are more concerned with the objects in a set being indiscernible. The key idea of using rough decomposition is to make the intrinsic indiscernibility of sampled computation part of the computation itself.

A fuzzy partition of Ω is a family (a_i) of N fuzzy sub-sets of Ω. Some properties are requested for a partitioning:

i) The (a_i) are supposed to cover Ω enough :

$$\operatorname*{Inf}_{\theta\in\Omega}\left\{\max_{i=1\ldots N}\left(\mu_{a_i}(\theta)\right)\right\}>0$$

ii) The (a_i) are discernible:

$$\operatorname*{Sup}_{\theta\in\Omega}\left\{\min\left(\mu_{a_i}(\theta),\mu_{aj}(\theta)\right)\right\}<1\ \forall(i,j)\in[1,N]^2$$

iii) The (a_i) provide a uniform partition:

$$\sum_{i=1\ldots N}\mu_{a_i}(\theta)=1\ \forall\theta\in\Omega.$$

Then, a rough decomposition of the fuzzy subset Θ on the fuzzy partition (a_i) is given by the mean of N pairs (Π_i,N_i) with:

$$\Pi_i(\Theta)=\Pi(\Theta|a_i)=\operatorname*{Sup}_{\theta\in\Omega}\left\{\min\left(\mu_\Theta(\theta),\mu_{a_i}(\theta)\right)\right\} \quad (6)$$

$$N_i(\Theta)=N(\Theta|a_i)=\operatorname*{Inf}_{\theta\in\Omega}\left\{\max\left(\mu_\Theta(\theta),1-\mu_{a_i}(\theta)\right)\right\} \quad (7)$$

Π_i can be seen as the degree of possible membership of (a_i) in Θ, while N_i is a degree of certain membership. Fig.1 illustrates such a concept on a triangular decomposition.

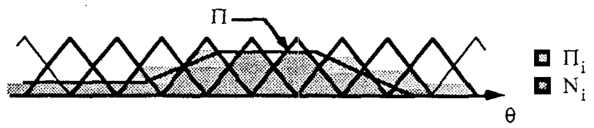


Fig.1: decomposition of a fuzzy set on a fuzzy partition.

Triangular decomposition is the most commonly used for different reasons. It is easy to compute. It satisfies all the expected properties (i,ii,iii), and it is the most neutral way of representing a sparse knowledge. A triangular set is a fuzzy number [DUB 81] with a linear form function. A triangular fuzzy number can be represented by two parameters: spread and mean.

Finally, a desirable property can be added:

$$\operatorname*{Inf}_{\theta\in\Omega}\left\{\max_{i=1\ldots N}\left(N(M_k|a_i)\right)\right\}>0$$

that limits the loss of information due to decomposition.

Some properties still hold on rough decomposition on a triangular fuzzy partition:

$$\Pi((P\cap Q)|a_i)=\min\left(\Pi(P|a_i),\Pi(Q|a_i)\right)$$

$$N((P\cup Q)|a_i)=\max\left(N(P|a_i),N(Q|a_i)\right)$$

But other properties don't:

$$\Pi((P\cup Q)|a_i)\leq\max\left(\Pi(P|a_i),\Pi(Q|a_i)\right)$$

$$N((P\cap Q)|a_i)\geq\min\left(N(P|a_i),N(Q|a_i)\right)$$

Because we aim at computing the upper and lower bounds of the distributions, if no equality is available we take the upper bound for possibility and the lower bound for necessity.

Then, the formulation of the filter becomes:

$$\begin{cases}\Pi(\Theta_k \cap M_{k+1}|a_i) = \min(\Pi(\Theta_k|a_i), \Pi(M_{k+1}|a_i)) \\ \Pi(\Theta_k \cup M_{k+1}|a_i) \approx \max(\Pi(\Theta_k|a_i), \Pi(M_{k+1}|a_i))\end{cases}$$

$$\text{and} \tag{8}$$

$$\begin{cases}N(\Theta_k \cap M_{k+1}|a_i) \approx \min(N(\Theta_k|a_i), N(M_{k+1}|a_i)) \\ N(\Theta_k \cup M_{k+1}|a_i) = \max(N(\Theta_k|a_i), N(M_{k+1}|a_i))\end{cases}$$

$$\Pi(D_{k+1}|a_i) = \min(\Pi(\Theta_k \cap M_{k+1}|a_i), 1-\eta_k) + \min(\min(\Pi(\Theta_k \cup M_{k+1}|a_i), 1-\eta_k), 1-N(\Theta_k|a_i))$$

$$\text{and} \tag{9}$$

$$N(D_{k+1}|a_i) = \min(N(\Theta_k \cap M_{k+1}|a_i), 1-\eta_k) + \min(\min(N(\Theta_k \cup M_{k+1}|a_i), 1-\eta_k), 1-\Pi(\Theta_k|a_i))$$

with

$$\eta = \Pi(\Theta_k|M_{k+1}) + N(\Theta_k|M_{k+1}) - \Pi(\Theta_k|M_{k+1}).N(\Theta_k|M_{k+1})$$

$$\begin{cases}\Pi(\Theta_k|M_{k+1}) \approx \sup\limits_{(i,j)=[1,N]^2}\left\{\min\begin{pmatrix}\Pi(\Theta_k|a_i), \\ \Pi(M_{k+1}|a_j), \\ \Pi(a_i|a_j)\end{pmatrix}\right\} \\ N(\Theta_k|M_{k+1}) \approx 1 - \sup\limits_{(i,j)=[1,N]^2}\left\{\min\begin{pmatrix}N(\Theta_k|a_i), \\ \Pi(M_{k+1}|a_j), \\ \Pi(a_i|a_j)\end{pmatrix}\right\}\end{cases}$$

IV. FUSING WITH GUESS FILTER

Usually, fusion of data or information from multiple sensors over time can take place at different levels of representation [LUO 92]. At signal level, statistical (or other) estimation processes are used to increase confidence or precision of the fused information. Most of these methods make very strong assumptions concerning to what degree the data are in registration. The fusion is therefore not robust because depending on how adequate the sensor model is. On the contrary, fusion at symbolic level works at the highest level of abstraction and is hence more robust to sensor modeling errors.

Consider now our experiment. The robot is equipped with two sensors, a compass and a gyrometer, with common measurement axes. The compass gives a measurement of the yaw angle while the gyrometer gives a measurement of its derivative. The compass gives an unbiased but very noisy measurement of the heading. Moreover, the data it provides become very unstable when the yaw angle varies. On the contrary, noise on gyrometer data does not depend on heading variations. However, it gives a highly biased estimation of the derivative because of its intrinsic thermal drift [BAR 93]. This drift is very difficult to model.

Using only compass data to estimate the heading will provide a delayed estimation when the vehicle moves. This is due to the fact that it is somewhat difficult for a Guess filter to distinguish between noise and a real variation, if this variation is smaller than the expected precision. Conversely, the short term estimation of the heading variation obtained by integration of the gyrometer data will be only slightly biased. However, a long term estimation would be subject to drift with time because of gyrometer drift and integration process. Therefore, there are two classical ways of improving estimation by fusion. The first solution acts at a low level, and consists of using a filter such as Kalman filter to obtain a weighted combination of gyrometer and compass estimations. The second solution acts at a high level and takes advantage of each estimation by selecting the most likely estimation by mean of a decisional process.

Guess filter provides a good framework to fuse information at both signal and symbolic level. Fusion using Guess filter is performed in three stages. First, a prediction of the expected possibility distribution is obtained by fuzzy addition. Then, some confidence parameter are computed quantifying symbolical concepts like coherence between derivative and evolution of the heading, a posteriori possibility and necessity, ... These confidence parameters are used to deduce two possibility distributions associated with two hypotheses which are here "the robot moves" and "the robot doesn't move". Then, the a posteriori distribution is computed by using a compromising fuzzy logical operator. Finally, this a posteriori distribution is used to update the a priori distribution. Only Θ_k is addressed here by this new inference procedure: $\partial\Theta_k$ is updated using the method given in III.

IV.1. PREDICTION

Θ_k (rsp. $\partial\Theta_k$) is the *a posteriori* possibility distribution of the heading (rsp. its derivative) at time k. Two hypotheses hold for the evolution model of the heading. If there is no motion, then $\hat{\Theta}_{k+1} \approx \Theta_k$; else $\hat{\Theta}_{k+1} = \Phi_{k+1} \approx \Theta_k + \partial\Theta_k$, Φ is the prediction in case of motion.

Θ_k is decomposed on a family (a_i) while $\partial\Theta_k$ is decomposed on (b_i). Computation of the prediction for the second hypothesis has to be achieved by fuzzy addition [DUB 81]. Fuzzy addition on fuzzy decomposition gives:

$$\Pi(\Phi_k|a_i) = \sup_{\theta\in\Omega}\left(\min\left(\mu_{\Theta_{k+\delta\Theta_k}}(\theta), \mu_{a_i}(\theta)\right)\right)$$

$$\leq \sup_{(m,n)}\left(\min\left(\Pi(\Theta_k|a_m), \Pi(\delta\Theta_k|b_n), \Pi(a_m + b_n|a_i)\right)\right)$$

$$\text{and} \tag{11}$$

$$N(\Phi_k|a_i) = \inf_{\theta\in\Omega}\left(\max\left(\mu_{\Theta_{k+\delta\Theta_k}}(\theta), 1-\mu_{a_i}(\theta)\right)\right)$$
$$\geq \inf_{(m,n)}\left(\max\left(N(\Theta_k|a_m), N(\delta\Theta_k|b_n), 1-\Pi(a_m+b_n|a_i)\right)\right)$$

$\Pi(a_m+b_n)$ has to be computed by advance and stored in a look up table in order to make the algorithm faster.

IV.2. INFERENCE

Let M_k be the measurement provided by the compass at time k. Possibility and necessity of inference of Θ_k and Φ_k with M_{k+1} are computed using method given in part III. Fuzzy tests are then used to estimate λ, that express the coherence between M_{k+1}, Θ_k and $\partial\Theta_k$. M_{k+1} is said to be coherent with the predicted values Θ_k and $\partial\Theta_k$ if \b($\partial\Theta_k \leq 0$) implies \b(($M_{k+1} \geq \Theta_k$) implies ($\Theta_k \geq M_{k+1}$)) and \b($\partial\Theta_k \geq 0$) implies \b(($M_{k+1} \leq \Theta_k$) implies ($\Theta_k \leq M_{k+1}$)).

$$\Pi(M_{k+1} \geq \Theta_k) = \sup_{u\geq v}\left\{\min\left(\mu_{M_{k+1}}(u), \mu_{\Theta_k}(v)\right)\right\}$$
$$\leq \sup_{i,j}\left\{\min\left(\Pi(M_{k+1}|a_i), \Pi(\Theta_k|a_j), \Pi(a_i \geq a_j)\right)\right\} \quad (12)$$

$$\Pi(\delta\Theta_k \geq 0) = \sup_{u\geq 0}\left\{\mu_{\delta\Theta_k}(u)\right\}$$
$$\leq \sup_{i}\left\{\min\left(\Pi(\delta\Theta_k|a_i), \Pi(a_i \geq 0)\right)\right\} \quad (13)$$

Then Gödel's implication (denoted $\rightarrow$) is used to compute λ.

$$\lambda = \min\left\{\begin{matrix}\Pi(\Theta_k \geq 0) \rightarrow (\Pi(M_{k+1} \geq \Theta_k) \rightarrow \Pi(\Theta_k \geq M_{k+1})), \\ \Pi(\Theta_k \leq 0) \rightarrow (\Pi(M_{k+1} \leq \Theta_k) \rightarrow \Pi(\Theta_k \leq M_{k+1}))\end{matrix}\right\} \quad (14)$$

λ is used to reduce the reliability of the observation. M_{k+1} is replaced by $M'_{k+1} = M_{k+1} \cap \Omega_\lambda$. ($\Omega_\lambda$ is the fuzzy subset of Ω such that $\forall u \in \Omega$, $\mu_{\Omega_\lambda}(u)=\lambda$). We obtain two possible intersections and unions :

$$I_{k+1} = (\Theta_k \cap M'_{k+1}) \cap \Omega_\eta$$
$$U_{k+1} = \left((\Theta_k \cup M'_{k+1}) \cap \overline{M'_{k+1}}\right) \cap \Omega_{1-\eta}$$
$$I'_{k+1} = (\Phi_k \cap M'_{k+1}) \cap \Omega_{\eta'}$$
$$U_{k+1} = \left((\Phi_k \cup M'_{k+1}) \cap \overline{M'_{k+1}}\right) \cap \Omega_{1-\eta'} \quad (15)$$

with

$$\eta = \Pi(\Theta_k|M_{k+1}) + N(\Theta_k|M_{k+1}) - \Pi(\Theta_k|M_{k+1}).N(\Theta_k|M_{k+1})$$
$$\eta' = \Pi(\Phi_k|M_{k+1}) + N(\Phi_k|M_{k+1}) - \Pi(\Phi_k|M_{k+1}).N(\Phi_k|M_{k+1})$$

IV.3. COMPROMISE

Now a compromise has to be found to update a priori possibility distribution Θ_k. This compromise uses the a priori reliability of each source of information at time k. In our experiments, reliability of the compass is linked to the fact that the robot doesn't move. Derivative estimation is used when the robot moves. The number σ represents the possibility of the robot to be moving. $\sigma = \Pi(\partial\Theta_k=0) = \min(\Pi(\partial\Theta_k \leq 0), \Pi(\partial\Theta_k \geq 0))$.

D_{k+1}, the deduction is obtained as follow:

$$D_{k+1} = \left[I_{k+1} \cup (I'_{k+1} \cap \Omega_{1-\sigma})\right] \oplus \left[U_{k+1} \cup (U'_{k+1} \cap \Omega_{1-\sigma})\right]$$

where $\oplus$ is the set operation such that

$$\mu_{A\oplus B}(u) = \min((\mu_A(u)+\mu_B(u)), 1)$$

Finally, Θ_{k+1} is computed using formula (5).

V. DEFUZZIFICATION.

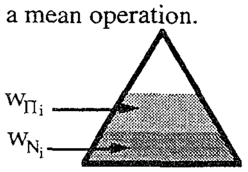
The use of such a filter in a real process implies the ability to provide one real crisp value for the obtained distribution. Such a transformation is also needed to allow some comparisons between this filter and other filters. This process is known in fuzzy control as "defuzzification".

For monomodal statistical processes, like Kalman filter, this transformation is trivial because the best estimate is part of the distribution representation. For multi-modal processes, even if those are statistical processes, this transformation is not problem-free.

The literature provides several ways of performing a defuzzification. Because the possibility distribution can be seen as a sparse knowledge, the main problem addressed here is to find a compromise between all these pieces of information. In general, the so-called barycenter method is used. This method consists of calculating the barycenter of the obtained fuzzy sets. However some problems arise. As a matter of fact, if the fuzzy set is made of the union of two disjoint intervals (e.g. $[21,25] \cup [29,31]$), this approach will provide a value (here 25.625) which does not belong to any fuzzy set (i.e. is not possible according to the knowledge). Conversely, the maximum method consists of finding the peak in the fuzzy subset. In case there is more than one peak, a mean value is provided or a selection rule is applied. This method has a flaw: it provides a highly biased value.

In order to avoid most of the drawbacks of each method, we use the method proposed in [DUB 88] consisting of associating a probability distribution with the given possibility distribution. This process is performed in three steps:

1) two weights are associated with each cell taking into account the shape of the cell. w_{Π_i} (rsp. w_{N_i}) is computed by using the conditional possibility (rsp necessity) of the cell. Then the weight is computed using a mean operation.



$$w_i = \mathcal{M}\left(w_{\Pi_i}, w_{N_i}\right)$$

Fig. 2: weight associated with each cell

2) the weights are sorted in order:

$$w'_i \geq w'_{i+1} \quad \text{and} \quad w'_{N+1} = 0$$

3) the probability distribution is then obtained by:

$$P_i = \sum_{k=i}^{N} \frac{1}{k}\left(w'_i - w'_{i+1}\right)$$

Then the probability is used to weight each cell to provide the estimate. This method can be improved by using robust statistics [HUB 81].

Finally, three measurements are available to qualify the fuzzy estimation: a confidence is given with the possibility and the necessity, and the fuzzy cardinal gives a measurement of the precision.

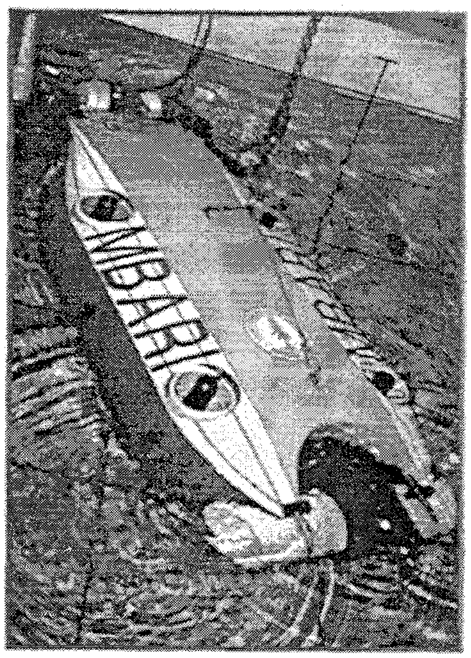
Because it works on hypothesis verifications at different levels, we called this filter: "Guess filter".

VI. EXPERIMENTAL RESULTS

This algorithm has been tested on simulated and real data. The examples below are real signals provided by a compass and a gyrometer mounted on the submarine vehicle OTTER[1]. Comparison result between Guess filter and Kalman filter have already been given in previous papers [DEV 93], [STR 96].

This example allows a qualitative comparison between Guess filtering using only compass signal and Guess filtering using fusion of compass' and gyrometer's signals. During the experiment, the robot is floating on the water in the tank. At time t≈0.6s, the robot is pushed suddenly on the right. This perturbation induces a great perturbation on the compass data. This perturbation cannot be rejected by guess filter without fusion. Using fusion, the guess filter is able to reject movement on the left and accept movement on the right because of the semantic link between derivative and position estimations.

[1] OTTER is a semi autonomous submarine vehicle developed by the Monterey Bay Aquarium Research Institute (California, USA).

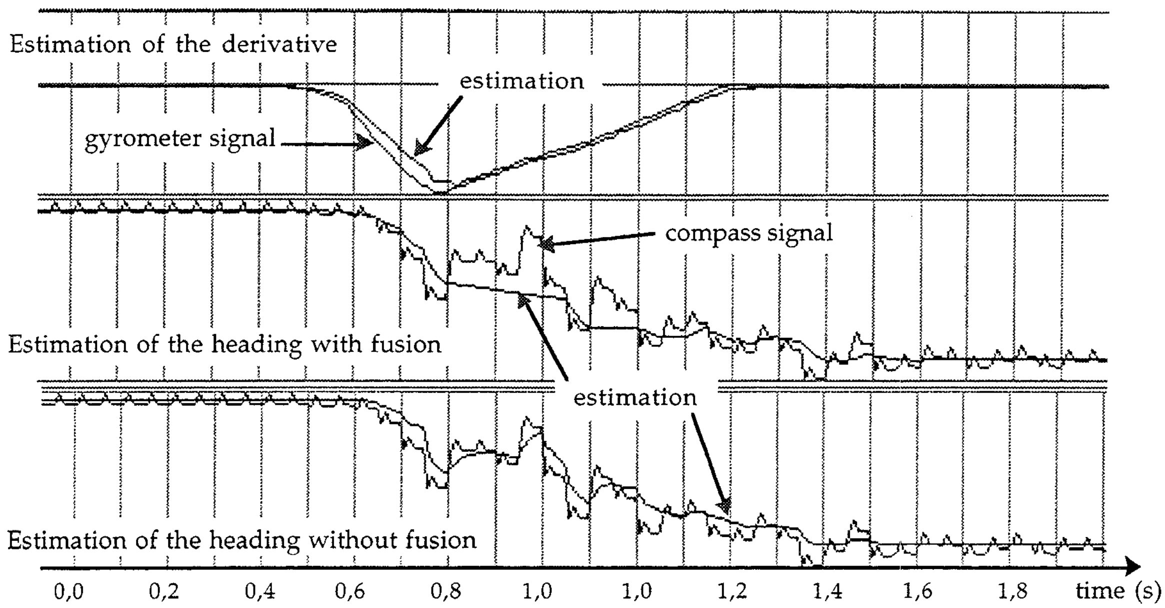


Semi-autonomous vehicle *OTTER*

VI. CONCLUSION

In this paper, we proposed to use Guess filtering to estimate the heading of a submarine robot by fusing compass and gyrometer data. Guess filtering is a new way of performing signal processing that uses three theories of error together: possibility-, statistical- and rough-sets theory. Possibility theory is a general enough framework to allow the representation of both uncertainty and imprecision in a quite simple manner. Rough sets deal with approximation due to computation and sampling. Using rough decomposition on fuzzy partition makes the error in computation part of the computation itself.

Using the Guess filter to perform sensor data fusion allows to combine data at both high and low level. Signal level fusion can be performed by using fuzzy addition. Semantic level fusion is used to select the most appropriate sensor using criteria and rules. In our experiment, only one rule has been implemented for sensor selection depending on how fast the robot moves. If the robot doesn't move or moves slowly, semantic inference prefers the compass estimation because it is less sensitive to bias. When the robot moves fast, the signal level process modifies possibility distribution of the heading, while semantic level process discards non-coherent compass data.



Several questions remain open. The first one is related to the defuzzification process. As a matter of fact, the weighting process we use try and find the "global movement". It consider that outliers can be discarded by probability assessment. We are currently working on a wavelet-based process searching for the "best" pike in the possibility distribution. This new process would be able to give more than one "most possible value" of the parameter to be estimated. Those values can be taken into account by a last high-level process dealing with security and risk to select the best value to be given to the control process. The second question concerns optimization of the computation of the filter. Particularly fuzzy addition is very time consuming. Finally, we aim to perform a fully estimation of the attitude of the robot by using more than six redundant sensors.

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